AMSC/CMSC 460: HW #4
Due: Tuesday 2/26/19 (in class)

Please submit the solution to at least one problem in LaTeX.

1. Assume that a and b are arbitrary constants. Show that every matrix of the form

$$A = \begin{pmatrix} 0 & 0 \\ a & b \end{pmatrix}$$

has an LU factorization. Does it have a Doolittle factorization?

2. Find a Doolittle factorization of

$$A = \begin{pmatrix} 1 & 5 \\ 3 & 15 \end{pmatrix}$$

(i.e., an LU factorization in which L is a **unit** lower triangular).

3. Verify that the following matrices are positive definite and find their Cholesky factorization:

i.

$$A = \begin{pmatrix} 4 & -1 & 1 \\ -1 & 3 & 0 \\ 1 & 0 & 2 \end{pmatrix}$$

ii.

$$A = \begin{pmatrix} 4 & 2 & 2 \\ 2 & 6 & 2 \\ 2 & 2 & 5 \end{pmatrix}$$

4. Use Matlab to find the Cholesky factorization of

i.

$$A = \begin{pmatrix} 4 & 0 & 2 & 1 \\ 0 & 3 & -1 & 1 \\ 2 & -1 & 6 & 3 \\ 1 & 1 & 3 & 8 \end{pmatrix}$$

ii.

$$A = \begin{pmatrix} 4 & 1 & 1 & 1 \\ 1 & 3 & 0 & -1 \\ 1 & 0 & 2 & 1 \\ 1 & -1 & 1 & 4 \end{pmatrix}$$

5. Determine the LU factorization of

$$A = \begin{pmatrix} 9 & 10 & 0 \\ 12 & 26 & 4 \\ 0 & 9 & 12 \end{pmatrix}$$

in which L is a lower triangular matrix with fours on its main diagonal.

6. If A has a Doolittle factorization, what is a simple formula for the determinant of A?

7. Use Gaussian elimination with scaled row pivoting to solve the following linear systems. Round all calculations to three digits after the decimal point.

i.

$$\begin{aligned} 2.12x_1 - 2.12x_2 + 51.3x_3 + 100x_4 &= \pi, \\ 0.333x_1 - 0.333x_2 - 12.2x_3 + 19.7x_4 &= \sqrt{2}, \\ 6.19x_1 + 8.2x_2 - 1.00x_3 - 2.01x_4 &= 0, \\ -5.73x_1 + 6.12x_2 + x_3 - x_4 &= -1. \end{aligned}$$

ii.

$$\begin{aligned} \pi x_1 + \sqrt{2}x_2 - x_3 + x_4 &= 0, \\ ex_1 - x_2 + x_3 + 2x_4 &= 1, \\ x_1 + x_2 - \sqrt{3}x_3 + x_4 &= 2, \\ -x_1 - x_2 + x_3 - \sqrt{5}x_4 &= 3. \end{aligned}$$