Midterm #1 – Solutions

1) a) $\lim_{x\to\infty} \frac{2x^2-x+1}{3+2x-x^2} = \lim_{x\to\infty} \frac{2-\frac{1}{x}+\frac{1}{x^2}}{\frac{3}{x^2}+\frac{2}{x}-1} = \frac{2}{-1} = -2$

divide by x^2 the numerator & the denominator.

All the terms of the form $\frac{a}{x}, \frac{a}{x^2} \xrightarrow{x\to\infty} 0$ (a is any constant)

b) $\lim_{x\to 1^+} \left[\log_{10} x + \frac{x^2-1}{x}\right] = \log_{10} 1 + \frac{1^2-1}{1} = 0.$

c) $$\lim_{x\to 0} \frac{(x-h)^2-(x+h)^2}{h} = \lim_{x\to 0} \frac{x^2-2hx+h^2-(x^2+2hx+h^2)}{h}$$

$$= \lim_{x\to 0} \frac{-4hx}{h} = -4x.$$

d) $$\lim_{x\to 0} \frac{e^{(x+1)}\left(\sqrt{x+1}-1\right)}{x}$$

$$= \lim_{x\to 0} \frac{e^{x+1}\left(\sqrt{x+1}-1\right)\left(\sqrt{x+1}+1\right)}{x\left(\sqrt{x+1}+1\right)} = \lim_{x\to 0} \frac{e^{x+1}\left(x+1-1\right)}{x\left(\sqrt{x+1}+1\right)}$$

$$= \lim_{x\to 0} \frac{e^{x+1}}{\sqrt{x+1}+1} = \frac{e}{2}.$$

(2) Every function on its own is a continuous function as a sum / difference of continuous functions.

$\Rightarrow$ $y(t)$ is continuous for $t<-3$, $-3<t<0$, $0<t<1$, $t>1$.

It remains to check the continuity at the interface points.

At $t=-3$ $\lim_{t\to -3^-} y(t) = \lim_{t\to -3^-} |t| = 3.$

$\lim_{t\to -3^+} y(t) = \lim_{t\to -3^+} t = -3$

$\Rightarrow$ $y(t)$ is discontinuous at $t=-3$.

At $t=0$ $\lim_{t\to 0^-} y(t) = \lim_{t\to 0^-} t = 0$

$\lim_{t\to 0^+} y(t) = \lim_{t\to 0^+} (e^t - 1) = 0$

$\Rightarrow$ $y(t)$ is continuous at $t=0$.

At $t=1$ $\lim_{t\to 1^-} y(t) = \lim_{t\to 1^-} e^t - 1 = e-1$

$\lim_{t\to 1^+} y(t) = \lim_{t\to 1^+} 2t^2+3 = 5$

$\Rightarrow$ $y(t)$ is discontinuous at $t=1$.

(3) (a) $2^{x^2-9/4} = 4^{2x} = (2^2)^{2x} = 2^{4x}$

$\Rightarrow x^2 - 9/4 = 4x$

$x^2 - 4x - 9/4 = 0$

The solutions will be the roots of this quadratic equation:

$$x_{1,2} = \frac{4 \pm \sqrt{4^2 - 4 \cdot 1 \cdot (-9/4)}}{2} = \frac{4 \pm \sqrt{25}}{2} = \frac{4 \pm 5}{2} =$$

$$= \begin{matrix} 9/2 \\ -1/2 \end{matrix}.$$

(b) $\log_x 3 + \log_x \frac{2}{3} = 1$

$\log_x 3 \cdot \frac{2}{3} = 1 \Rightarrow \log_x 2 = 1 \Rightarrow x = 2.$

(4). $y(t) = 2\cos(t + \frac{3}{4}\pi) + 1.$

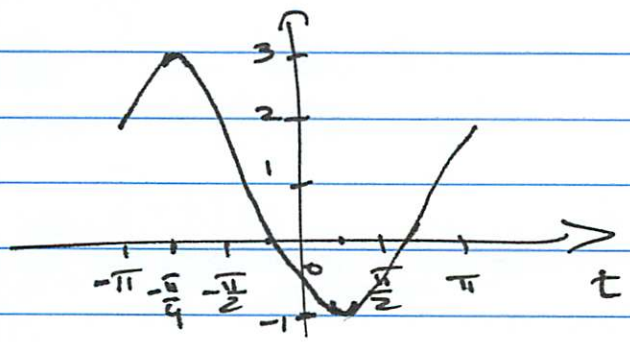
(a) The amplitude $= 2.$

(b) The period $= 2\pi.$

(c) Phase shift $= -\frac{3}{4}\pi$, i.e. $\frac{3}{4}\pi$ to the left.

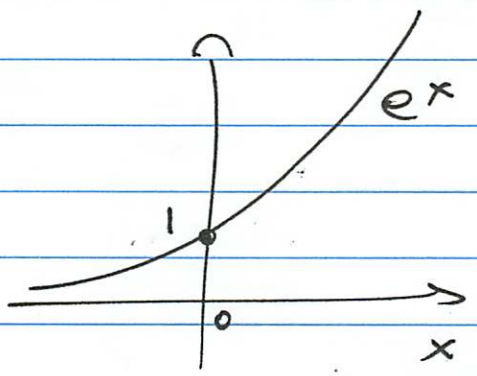
(d) Vertical shift $= 1$ upward.

(e)

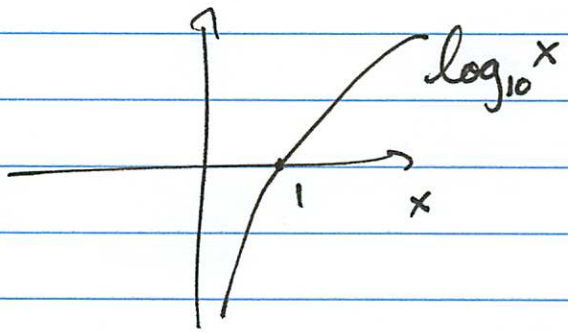


(5)

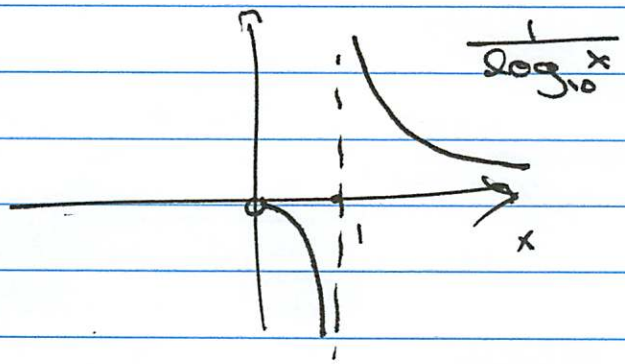
(a)


e^x
1
0
x

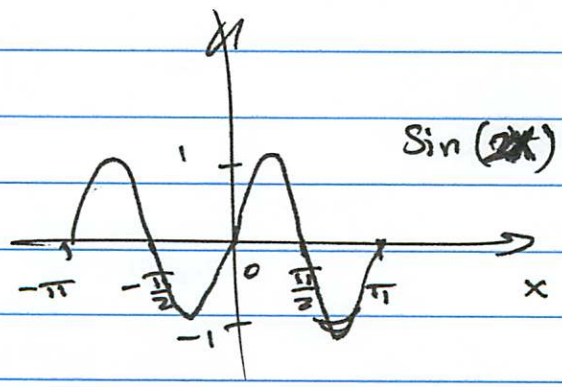
(b)


$\log_{10} x$
1
x

(c)


$\frac{1}{\log_{10} x}$
1
x

(d)


Sin (2x)
1
$-\pi$
$-\frac{\pi}{2}$
0
$\frac{\pi}{2}$
π
x
-1