Math 464: Final Exam
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Solve all problems. You may use 3 cheat sheets. You may not use calculators.

Start each problem on a new page and show your work.

Good luck!

1. (a) **(10 points)** Find the Fourier transform of the function

   $$f(x) = \int_{-1/2}^{1/2} e^{-\pi(x-u)^2} du.$$

   Hint: Write $f(x)$ as a convolution of 2 functions.

   (b) **(10 points)** Let $f_0(x) = e^{-x^2}$. Use the Fourier transform to compute the convolution $f_0 * f_0$.

2. (a) **(20 points)** Compute the Fourier coefficients and expand in Fourier series the following 1-periodic function

   $$f(x) = \begin{cases} x, & 0 < x < \frac{1}{2} \\ a, & x = \frac{1}{2} \\ 0, & \frac{1}{2} < x < 1 \end{cases}$$

   (b) **(5 points)** What should be a so that the Fourier series converges to $f(x)$ for every $0 < x < 1$.

3. **(25 points)** Solve the following initial-boundary value problem for the heat equation:

   $$u_t = 9u_{xx}, \quad 0 \le x \le 1$$
   $$u_x(0,t) = u_x(1,t) = 0$$
   $$u(x,0) = \cos(4\pi x)$$

   Note that the boundary conditions are for the derivative of u and not u.

4. Compute the Fourier transform of the following generalized functions

   (a) **(15 points)** $f(x) = \cos(2\pi x)$

   (b) **(15 points)** $f(x) = x \cdot \delta'(x)$