Math 464: HW #1
Due: Tuesday 2/3/15 (Submit by email to grader)

Many of the homework problems in this problem set are taken from Brown & Churchill, Complex Variables and Applications

1. The LaTex problem on the webpage.

2. In each case, sketch the set of points determined by the given condition

   (a) $|z - 2 - i| = \frac{1}{2}$
   (b) $|z - 2i| \leq 3$
   (c) $|z + 3i| \geq 5$
   (d) $\mathrm{Re}(\bar{z} - i) = 3$

3. Show that

   (a) $\frac{1+2i}{3-4i} + \frac{2-i}{5i} = -\frac{2}{5}$
   (b) $\frac{5i}{(1-i)(2-i)(3-i)} + \frac{1}{2} = 0$
   (c) $\frac{1}{4}(1 - i)^4 + 1 = 0$

4. Show that $|\mathrm{Re}(2 + \bar{z} + z^3)| \leq 4$ when $|z| \leq 1$.

5. Show that

   (a) $\mathrm{Arg}\left(\frac{i}{-2-2i}\right) = -\frac{3\pi}{4}$
   (b) $\mathrm{Arg}((\sqrt{3} - i)^6) = \pi$

6. By writing the individual factors on the left in exponential form, performing the needed operations, and changing back to rectangular coordinates, show that

   (a) $2^{11}(1 + \sqrt{3}i)^{-10} = (-1 + \sqrt{3}i)$
   (b) $i(1 - \sqrt{3}i)(\sqrt{3} + i) = 2(1 + \sqrt{3}i)$

7. Establish the identity

$$1 + z + z^2 + \ldots + z^n = \frac{1 - z^{n+1}}{1 - z}, \qquad z \neq 1,$$

   and then use it (by substituting $z = e^{i\theta}$) to derive Lagrange's trigonometric identity:

$$1 + \cos\theta + \cos 2\theta + \ldots + \cos n\theta = \frac{1}{2} + \frac{\sin[(2n + 1)\theta/2]}{2\sin(\theta/2)}, \qquad 0 < \theta < 2\pi.$$

8. Find all the roots (expressing them in rectangular coordinates) of $7^{1/7}$.

9. Compute the following integrals:

   (a) $\int_{-\pi}^{\pi} e^{-2it} dt$

   (b) $\int_0^1 e^{4\pi ix} dx$

   (c) $\int_0^1 e^{-\pi iw} e^{8\pi iw} dw$

   (d) $\int_0^1 e^{-2\pi int} dt$, where n is an integer. Remember to treat separately the case $n = 0$.