2.3 (a) ✗ Total: 16

3

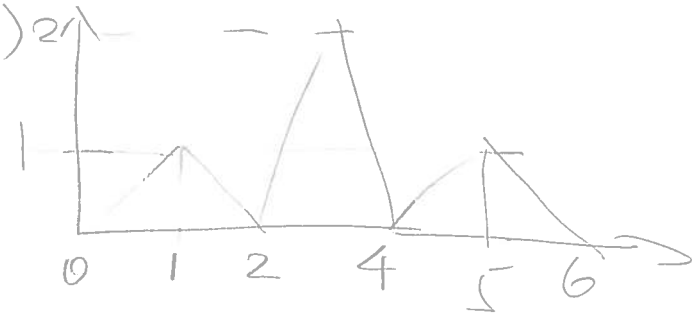
2.1 (a) $\int_0^1 g(x-u)\,du$

$0 < x-u < 1$

$\Rightarrow x-1 < u < x$

$\int_{x-1}^{x} du = 1$.

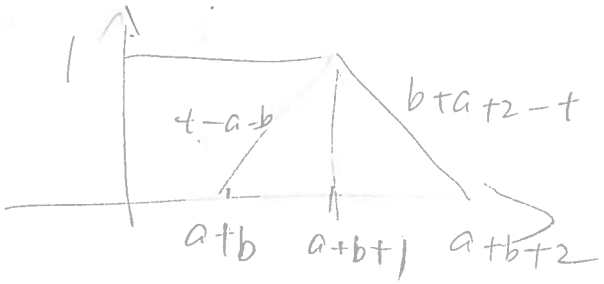
(d)



(f)

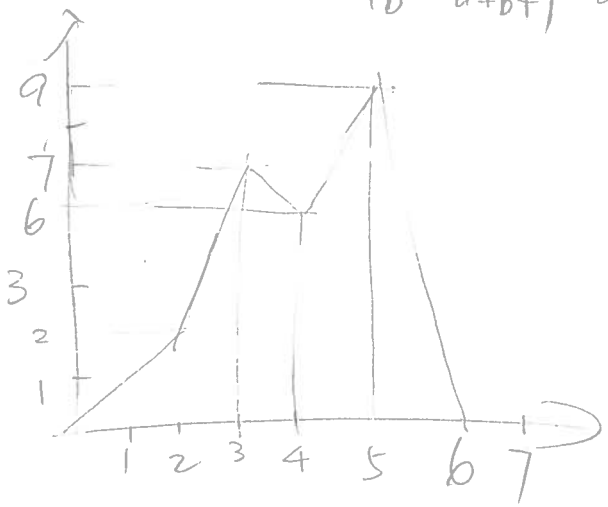
$t-a-1 < b < t-a$

$a+b < t < a+b+1$: $\int_b^{t-a} 1\,dt = t-a-b$



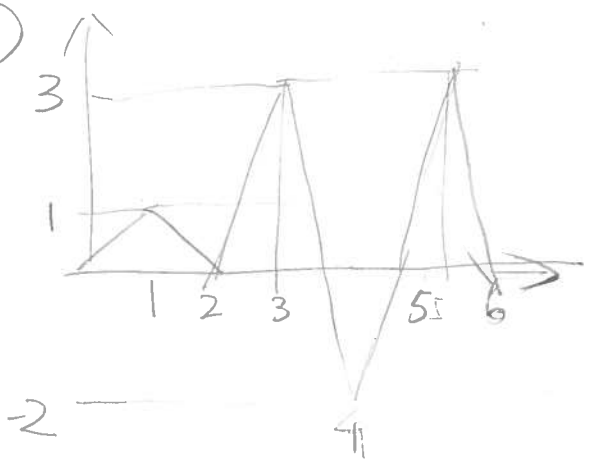
3

2.2 (a)

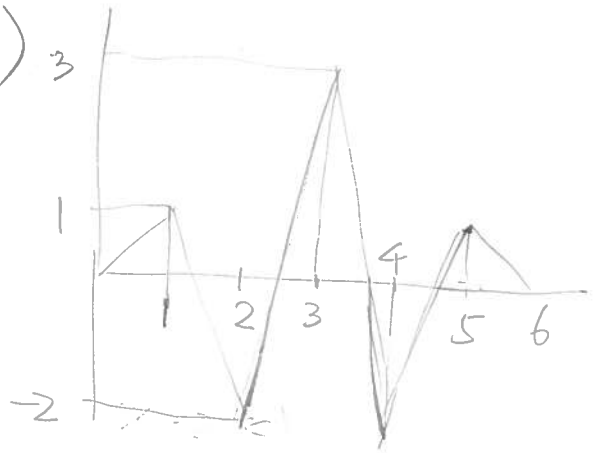


$$\begin{cases} x & 0<x<2 \\ 5x-8 & 2<x<3 \\ 10-x & 3<x<4 \\ 3x-6 & 4<x<5 \\ 54-9x & 5<x<6 \\ 0 & \text{--} \end{cases}$$

(b)



(d)

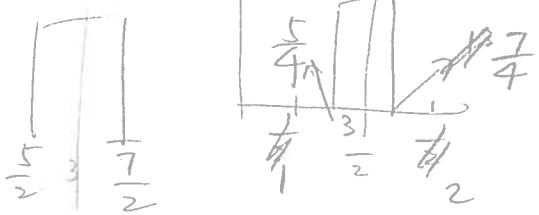


2.3 (a) $\begin{cases} x & x>0 \\ 0 & \end{cases}$

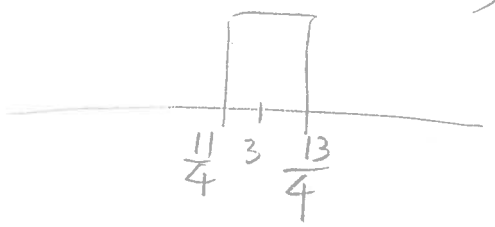
2.6 ~~(a)~~

Ex 3.1 (a) $\Pi(2(x-3/2))$

$\Pi(x-3)$



Ex 3.1 (b) $\Pi(2(x-3))$



(c) $\mathcal{F}\left(\Pi(2(x-\frac{3}{2}))\right) = e^{\frac{-3\pi i s}{2}} \frac{1}{2}\operatorname{sinc}\left(\frac{s}{2}\right)$

$\mathcal{F}(\Pi(2(x-3))) = \dfrac{e^{-6\pi s i}}{\frac{1}{2}\operatorname{sinc}\left(\frac{s}{2}\right)}$

$\Pi(x)$ has $F(t)$ $\operatorname{sinc}(s)$

$g(x) := f(x-x_0)$ has FT

$e^{-2\pi i s x_0} \cdot F(s)$ + translation rule

$g(x) := f(ax)$ has the FT

$G(s) = \frac{1}{|a|} F\left(\frac{s}{a}\right)$ dilation rule

$\frac{1}{2} e^{-\frac{6\pi i s}{2}} \operatorname{sinc}\left(\frac{s}{2}\right)$

$= \frac{1}{2} e^{-3\pi i s} \operatorname{sinc}\left(\frac{s}{2}\right)$

$= \frac{1}{2} e^{-3\pi i s} \dfrac{\sin\left(\frac{\pi s}{2}\right)}{\frac{\pi s}{2}}$

$= \dfrac{e^{-3\pi i s}}{\pi s} \sin\left(\frac{\pi s}{2}\right)$

3.1 (d) ~~$\mathcal{F}(b_1(x)) =$~~

$$\mathcal{F}(b(x)) = \frac{1}{2}\,\mathrm{sinc}\left(\frac{s}{2}\right)$$

$$\mathcal{F}(b_1(x)) = \mathcal{F}\left(b\left(x-\tfrac{3}{2}\right)\right) = \frac{1}{2}\,e^{-3\pi i s}\,\mathrm{sinc}\left(\frac{s}{2}\right)$$

$$\mathcal{F}(b_2(x)) = \frac{1}{2}\,e^{-6\pi i s}\,\mathrm{sinc}\left(\frac{s}{2}\right)$$

$$\mathcal{F}\left(b_1\left(x-\tfrac{3}{2}\right)\right) = \frac{1}{2}\,e^{-6\pi i s}\,\mathrm{sinc}\left(\frac{s}{2}\right)$$

3.2 (a) $\mathcal{F}\left(-\Pi(x+1)+\Pi(x-1)\right)$

$$= -e^{2\pi i s}\,\mathrm{sinc}(s) + e^{-2\pi i s}\,\mathrm{sinc}(s)$$

$$= \left(-e^{2\pi i s} + e^{-2\pi i s}\right)\mathrm{sinc}(s)$$

$$= \frac{-2i\sin(2\pi s)\,\sin(\pi s)}{\pi s}$$

$$= \frac{1}{\pi i s}\left[\cos(\pi s) - \cos(3\pi s)\right]$$

(c) $\mathcal{F}\left[\cos(4\pi x)\cdot\Pi(x)\right]$

$=$ $\dfrac{\sin[\pi i(2-s)]}{2\pi(2-s)}$ $+$ $\dfrac{\sin[\pi i(s+2)]}{2\pi(s+2)}$

$\mathrm{FT}(\cos 2\pi(2x)) = \frac{1}{2}\delta(s+2) + \frac{1}{2}\delta(s-2)$

$\mathrm{FT} = \frac{1}{2}\left(\mathrm{sinc}(s+2) + \mathrm{sinc}(s-2)\right)$

3.2 (e) $f(x) = x \cdot \Pi(x)$

power scaling rule

$g(x) = x \cdot f(x)$ has the FT $G(s) = (-2\pi i)^{-1} F'(s)$

$$\frac{1}{-2\pi i} \frac{d}{ds}\left(\frac{\sin(\pi s)}{\pi s}\right)$$

$$= -\frac{1}{2\pi i} \frac{\pi\cos(\pi s)(\pi s) - \pi\sin(\pi s)}{(\pi s)^2}$$

$$= -\frac{1}{2\pi i}\left(\frac{\cos \pi s}{s} - \frac{\sin(\pi s)}{\pi s^2}\right)$$

(g) $\Pi(2x) \cdot \Pi(3x)$

$= \Pi\Pi(3x)$

$\frac{1}{3}\operatorname{sinc}\left(\frac{s}{3}\right)$ or $\frac{1}{\pi s}\sin\left(\frac{\pi s}{3}\right)$