MATH 464 HW 4

3.9 ① 1 pt. (b) $\Lambda(2x+1)=0$ outside $(-1,0)$

$\Lambda(2x-1)=0$ outside $(0,1)$

$(-1,0)\cap(0,1)=0 \Rightarrow f(x)=\Lambda(2x+1)\cdot\Lambda(2x-1)=0$.

(d) ①

$\operatorname{sinc}(x)$ has the FT $G(s)=\Pi(s)$

$\operatorname{sinc}(4x)$ has the FT $\frac{1}{4}\Pi\left(\frac{s}{4}\right)$

$\operatorname{sinc}^2(x)$ has the FT $G(s)=\Lambda(s)$

$\operatorname{sinc}^2(2x)$ has the FT $G(s)=\frac{1}{2}\Lambda\left(\frac{s}{2}\right)$

FT of $f(x)$ is $\frac{1}{4}\Pi\left(\frac{s}{4}\right)\cdot\frac{1}{2}\Lambda\left(\frac{s}{2}\right)=\frac{1}{8}\Pi\left(\frac{s}{4}\right)\cdot\Lambda\left(\frac{s}{2}\right)$

$=\frac{1}{8}\Lambda\left(\frac{s}{2}\right)$ (since $\Pi\left(\frac{s}{4}\right)=1$ on $(-2,2)$)

(f) ①

$$\frac{(-2\pi i)(-2\pi i s)^2}{2!}e^{2\pi i * i s}\,h(s)$$

$$=4\pi^3 i\, e^{-2\pi s}\,h(s)$$

(h) ①

FT of $e^{-|x|}$ is $\frac{2}{1+4\pi^2 s^2}$

FT of $e^{-2|x+\frac{5}{2}|}$?

FT of $e^{-|x+\frac{5}{2}|}$ is $e^{-2\pi i s\cdot\left(\frac{5}{2}\right)}\frac{2}{1+4\pi^2 s^2}$

$$=e^{-5\pi i s}\frac{2}{1+4\pi^2 s^2}$$

FT of $e^{-2|x+\frac{5}{2}|}$ is $\frac{e^{-\frac{5\pi i s}{2}}}{1+4\pi^2\left(\frac{s}{2}\right)^2}=\frac{e^{-\frac{5\pi i s}{2}}}{1+\pi^2 s^2}$

(j) $f(x) = -\frac{i}{2}\frac{1}{x+1-i} + \frac{i}{2}\frac{1}{x+1+i}$ ①

$$\text{FT of } f(x) = -\frac{i}{2}(2\pi i)e^{2\pi i(1-i)s}h(-s)$$
$$+\frac{i}{2}(-2\pi i)e^{2\pi i(1+i)s}h(s)$$
$$= \pi e^{2\pi(1+i)s}h(-s) + \pi e^{2\pi(i-1)s}h(s)$$

3.14 (a) $F_1 = \Pi\left(\frac{s}{2}\right) \quad F_2 = \Pi\left(\frac{s}{4}\right)$ ①

$$F_1 \cdot F_2 = \Pi\left(\frac{s}{2}\right) \Rightarrow f_1 * f_2 = 2\,\mathrm{sinc}(2x)$$

(b) $F_1 = \Pi\left(\frac{s}{2}\right) \quad F_2 = \Lambda(s)$ ①

$$F_1 \cdot F_2 = \Lambda(s) \quad \left(\text{since } \Pi\left(\tfrac{s}{2}\right) = 1 \text{ on } (-1,1)\right)$$
$$\Rightarrow f_1 * f_2 = \mathrm{sinc}^2(x)$$

(c) $F_1 = e^{-\pi s^2} \quad F_2 = e^{-\pi s^2} \quad F_1 \cdot F_2 = e^{-2\pi s^2} = e^{-\pi(\sqrt{2}s)^2}$ ①

$$\Rightarrow f_1 * f_2 = \frac{1}{\sqrt{2}}e^{-\pi\left(\frac{x}{\sqrt{2}}\right)^2} = \frac{1}{\sqrt{2}}e^{-\frac{\pi x^2}{2}}$$

(d) $F_1 = F_2 = e^{-|s|} \quad F_1 \cdot F_2 = e^{-|2s|}$ ①

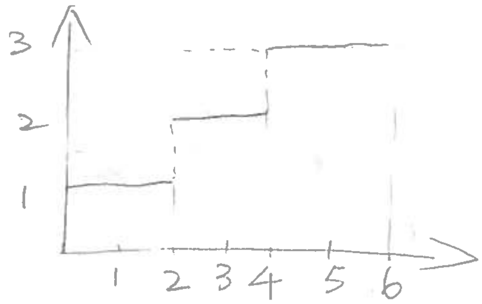
$$\Rightarrow f_1 * f_2 = \frac{1}{2}\frac{2}{1+4\pi^2\left(\frac{x}{2}\right)^2} = \frac{1}{1+\pi^2x^2}$$

(e) $F_1 = -h(-s)e^{s} \quad F_2 = h(s)e^{-s}$ ①

$$F_1 \cdot F_2 = -h(-s)h(s) = -\delta(s) \quad \text{or} \quad 0$$
$$\Rightarrow f_1 * f_2 = -1. \quad \text{or} \quad 0$$

3.20

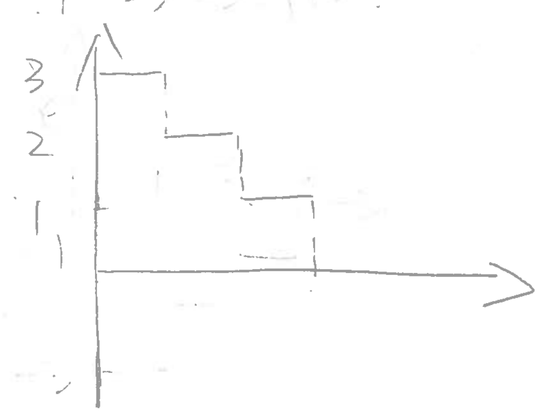
(b) FT of $2f(2x)$ is $F(\frac{s}{2})$

① 

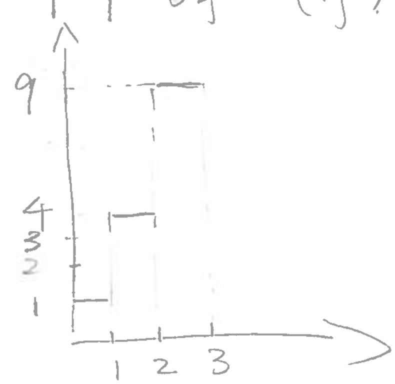
(d) FT of $e^{6\pi i x} f(-x)$ is $-F(-s)$

①

$F(-(s-3))$



(f) FT of $(f*f)(x)$ is $F^2(s)$

① 

(h) FT of $f(x)\cdot \mathrm{sinc}(x)$ is $F(s) * \Pi(s)$

② 2 pts.

$$= \int_{u-\frac{1}{2}}^{u+\frac{1}{2}} F(s)\, ds$$

$u+\frac{1}{2} \le 0 \iff u \le -\frac{1}{2}$ $\quad F(s) * \Pi(s) = 0$

$0 < u+\frac{1}{2} \le 1 \Rightarrow -\frac{1}{2} < u \le \frac{1}{2}$ $\quad \int_0^{u+\frac{1}{2}} 1\, ds = u+\frac{1}{2}$

$1 < u+\frac{1}{2} \le 2 \Rightarrow \frac{1}{2} < u \le \frac{3}{2}$ $\quad \int_{u-\frac{1}{2}}^{1} 1\, ds + \int_1^{u+\frac{1}{2}} 2\, ds = u+\frac{1}{2}$

$2 < u+\frac{1}{2} \le 3 \iff \frac{3}{2} < u \le \frac{5}{2}$ $\quad \int_{u-\frac{1}{2}}^{2} 2\, ds + \int_2^{u+\frac{1}{2}} 3\, dS = u+\frac{1}{2}$

$3 < u+\frac{1}{2} \le 4 \iff \frac{5}{2} < u \le \frac{7}{2}$ $\quad \int_{u-\frac{1}{2}}^{3} 3\, dS$

$= 3(3-u+\frac{1}{2}) = \frac{21}{2} - 3u$