Math 464: Midterm Exam #2 – Solutions
Prof. Doron Levy
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1. **(25 points)** Let

$$f(x) = \Lambda\left(\frac{x}{p/2}\right), \qquad g(x) = \sum_{m=-\infty}^{\infty} \Lambda\left(\frac{x-mp}{p/2}\right).$$

   (a) **(5 points)** Sketch the graphs of $f(x)$ and $g(x)$

   (b) **(8 points)** What is $F(s)$, the Fourier transform of $f(x)$?

   (c) **(12 points)** Use Poisson's relation to find the Fourier series of $g(x)$.

   Solution:

$$F(s) = \frac{p}{2}\text{sinc}^2\left(\frac{ps}{2}\right)$$

$$g(x) = \frac{1}{2}\sum_{k=-\infty}^{\infty} \text{sinc}^2\left(\frac{k}{2}\right) \exp\frac{2\pi ikx}{p}$$

2. **(25 points)** Consider the following function $f(x)$ on $\mathbb{T}_p$

$$f(x) = x, \qquad -p/2 < x < p/2.$$

   Compute the Fourier series of the p-periodic function $f(x)$ directly from the definition of the Fourier series.

   Solution: Page 175.

3. **(25 points)** Let

$$f(x) = e^{-|x|}.$$

   (a) **(12 points)** Show that $f''(x) = f(x) - 2\delta(x)$

   (b) **(13 points)** Use part (a) to compute $F(s)$, the Fourier transform of $f(x)$.

   Solution: Page 423

4. Verify the following identities by showing that the Fourier transform of both sides of the equation are equal.

   (a) **(12 points)** $\delta^{(m)}(x) * \delta^{(n)}(x) = \delta^{(m+n)}(x)$.

   Solution:

$$(2\pi is)^m \cdot (2\pi is)^n = (2\pi is)^{m+n}$$

(b) **(13 points)** $x\delta'(x) = -\delta(x)$.

Solution:

$$\delta'\hat{} = 2\pi i s \delta\hat{} = 2\pi i s$$

Hence

$$(x\delta)\hat{} = (-2\pi i)^{-2}(\delta\hat{})' = -1,$$

which is the Fourier transform of $-\delta(x)$.