

**Math 464: Midterm Exam #1**  
**Prof. Doron Levy**  
**February 27, 2014**

Solve all 4 problems.

1. **(25 points)** Find the Fourier transform of

$$g(x) = \frac{2}{x^2 - 4x + 5}.$$

2. **(25 points)** Compute

$$\int_{-\infty}^{\infty} \left( \frac{\sin(ax)}{x} \right)^4 dx, \quad a > 0.$$

3. **(25 points)** Find a function  $f(x)$  that satisfies

$$\int_{-\infty}^{\infty} f(u)f(x-u)du = e^{-\pi x^2}$$

4. Consider the differential equation  $-f''(x) + f(x) = e^{-2|x|}$  with  $x \in \mathbb{R}$ .

(a) **(13 points)** Find  $F(s)$ , the Fourier transform of  $f(x)$ .

(b) **(12 points)** Find constants  $A$  and  $B$  such that  $F(s)$  from part (a) can be written as

$$F(s) = \frac{A}{1 + 4\pi^2 s^2} + \frac{B}{1 + \pi^2 s^2}.$$

Use this expansion to find a solution to the given differential equation,  $f(x)$ .