Math 464: Midterm Exam #2
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Solve all 4 problems.

1. **(25 points)** Let
$$f(x) = \Lambda\left(\frac{x}{p/2}\right), \qquad g(x) = \sum_{m=-\infty}^{\infty} \Lambda\left(\frac{x-mp}{p/2}\right).$$
   (a) **(5 points)** Sketch the graphs of $f(x)$ and $g(x)$
   (b) **(8 points)** What is $F(s)$, the Fourier transform of $f(x)$?
   (c) **(12 points)** Use Poisson's relation to find the Fourier series of $g(x)$.

2. **(25 points)** Consider the following function $f(x)$ on $\mathbb{T}_p$
$$f(x) = x, \qquad -p/2 < x < p/2.$$
   Compute the Fourier series of the p-periodic function $f(x)$ directly from the definition of the Fourier series.

3. **(25 points)** Let
$$f(x) = e^{-|x|}.$$
   (a) **(12 points)** Show that $f''(x) = f(x) - 2\delta(x)$
   (b) **(13 points)** Use part (a) to compute $F(s)$, the Fourier transform of $f(x)$.

4. Verify the following identities by showing that the Fourier transform of both sides of the equation are equal.
   (a) **(12 points)** $\delta^{(m)}(x) * \delta^{(n)}(x) = \delta^{(m+n)}(x)$.
   (b) **(13 points)** $x\delta'(x) = -\delta(x)$.