

Combinatorial analysis.

1. *On a test two topics out of five will be tested. Bob knows three of them. If both topics on the test are familiar to him he gets A, one-B, non-D. Compute the probability of each grade.*
2. *A box contains 5 blue balls numbered from 1 to 5 and 3 green balls numbered from 1 to 3. How many ways are there to select one blue and one green ball from this box?*
3. *In a parliament of a certain country there are four parties having 30, 20, 15 and 10 representatives respectively. For a certain committee one member should be delegated from each party. How many different committees are possible?*
4. *Alice and Bob want to split. Their household has 20 valuable objects. How many ways are there to divide them?*
5. *5 friends have 5 theater tickets. How many ways are there to distribute the tickets among them?*
6. *5 couples have theater tickets. How many ways to sit them are possible if each couple wants to sit together?*
7. *Bob wrote checks to his water company, gas company, electric company, trash collectors and two credit card companies. Being absent minded he put these checks randomly to 6 envelopes provided. What is the probability that all his creditors got the right checks?*
8. (a) *4 men and 4 women attend a dinner. If a table is round and men and women should alternate, how many seating arrangements are possible?*
(b) *The same question if it is only important who sits next to whom, not which particular seats they occupy.*
(c) *The same question if Alice does not want to sit next to Bob.*
9. (a) *Bob has 14 bottles of wine. He has to serve 4 for a dinner party. How many choices does he have?*
(b) *Bob has 8 bottles of red wine and 6 bottles of white wine. He has to serve 2 reds and 2 whites for a dinner party. How many choices does he have?*
10. *During the month of September John, Jack and Jill have to work 20 evening shifts each so that each day two of them are present. How many different schedules are possible?*
11. (a) *How many different license plates with three letters and three digits following the letters are possible?*

(b) How the answer changes if the order can be arbitrary (i.e. letters and numbers can be intermixed)?

12. A certain department has 20 people. They have to choose 4 committees. Each committee has a chairman, a secretary and a regular member.

(a) How many choices are possible?

(b) The same question if no person can serve on two committees?

(c) The same question if the only restriction is that all committees should be different.

(d) Do part (b) if we only have to choose each committee's personnel leaving duty assignment inside the committee to the members?

13. How many different letter arrangements can be made from the letters NONSENSE?

14. During the day Bob eats 4 oranges, 3 apples and 7 cookies. How many different orders are possible?

15. During the month of August Bob has to work three night shifts. How many ways to choose the nights are there if he can not work two nights in a row?

16. Expand $(2x + 1)^4$, $(x + y)^5$.

17. (a) A fair coin is tossed 19 times. What is the probability that the number of heads is even?

(b) Same for 20 tosses.

18. (a) Alice has painted 20 pictures. She needs to send 5 pictures each to 3 different exhibitions. How many choices does she have?

(b) The same question if 10 of her pictures are from the black period and 10 are from the orange period and she wants each exhibition to show 2 black and 3 orange pictures.

(c) The same question if the only restriction is that, overall, more orange pictures should be exhibited?