

### Conditional probability. Independence.

1. *Eleanor and Felix are playing bridge. If Eleanor has 2 aces what is the probability that Felix has at least one ace? (There are 4 plays getting 13 cards each.)*
2. *Among cars of a certain make 20 % experience the first mechanical problem during the first year, 20 % during the fifth year, 30 % during the tenth year and 30 % during the fifteenth year. If the car survives the first year what is the probability that it will run without problems for 8 more years?*
3. *There are three cards. The first colored red on both sides, the second is colored red on one side and blue on another and the third card is colored blue on both sides. A card is drawn at random. If one of its sides is colored red what the other side is colored red?*
4. *A coins was tossed 10 times. Given that there were 6 heads and 4 tails what is the probability that the first toss was a head? first three tosses has at least one tail?*
5. *A box contains 5 blue, 5 green and 5 red balls. The balls are taken from the box in a random order (without replacement). What is the probability that the first ball is red, the second is blue and the third is green?*
6. *An urn contains 1 black ball and 1 white ball. At each step one ball is chosen from the urn at random and returned together with another ball of the same color. Find the probability that after 998 trials there are at least 700 white balls in the urn.*
7. *A box contains 7 blue and 3 red balls. If the balls are taken in a random order what is the probability that the second ball is red? the third ball is red?*
8. *Eleanor and Felix are tossing a coin. If it is a head Felix gives to Eleanor \$ 1 otherwise she gives \$ 1 to him. The game continues until one of them loose all the money. If initially Felix has \$ 10 and Eleanor has \$ 15 what is the probability that Eleanor will win?*
9. *At a certain show there are three doors. Two are empty and one contains prize. The player chooses a door. The host opens one of the two remaining doors which is empty (if both are empty she opens either with probability  $1/2$ ). The player then has an option to switch doors or stick to her initial choice. What is the probability that the prize is behind the other door?*

**10.** *At a competition some athletes are chosen for a random drug test. If an athlete uses drugs she is caught with probability 90 % and if she does not use drugs the test gives positive result in 1 % of cases. 1 in 100 athletes in this sport uses drugs before competitions having tests. If an athlete tests positive what is the probability that he used drugs?*

**11.** *Events  $A, B$  and  $C$  are independent. If  $P(A) = \frac{1}{2}$ ,  $P(B) = \frac{1}{3}$ ,  $P(C) = \frac{2}{3}$  what is the probability that exactly two events occur? all three events occur?*

**12.**  $p = \frac{2}{3}$

**13.** *A die is rolled several times. What is the probability that 1 occurs before an even number?*

**14.** *Eleanor and Felix have the following bet. They roll a die. If the outcome is 1,2,3,4 then Felix wins, otherwise Eleanor wins. Felix bets what he would win 4 times before Eleanor wins twice. Should she accept this bet?*

**15.** *In a certain country there are two factories melting coins (each producing about 50 % of the coin supply). A coin made by factory  $A$  lands on head 60 % of the time and a coin made by factory  $B$  lands on head 40 % of the time. A random coin is tossed three times resulting in 2 heads and 1 tail. What is the probability that the next toss will be a head?*

**16.** *In a probability class 25 % of students can solve 90 % of the problems, 25 % of students can solve 80 % of the problems, 25 % of students can solve 70 % of the problems, and 25 % of students can solve 10 % of the problems. Three random problems are chosen for the test. If the student solved two of them what is the probability that he can solve 90 % of all problems?*