Discrete Random Variables. Expectation. Variance.

1. *A certain class has 7 male and 5 female teaching assistants. 5 people are chosen randomly to proctor an exam. Find the probability that majority of them are women.*

2. *A die is rolled 6 times. Let X be the maximal result. Compute $P(X = i)$ for $i = 1 \dots 6$.*

3. *Joan bought 7 boxes of cereal. Each box contains a picture of either dog, cat, fish or bird. A person having a complete set of pictures can have a free cereal box.*

(a) What is the probability that Joan can have a free box?

(b) What is the probability that she has exactly 3 different pictures?

4. *Let $p(x) = c\left(\frac{1}{3}\right)^x$, for $x = 0, 1, 2 \dots$ Find c.*

5. *(a) A lottery ticket costs \$1. 4 numbers are selected randomly from a set of 20 (the order is not important). If you have guessed all numbers correctly you get \$4000. You will also get \$10 for three correct answers. If you buy one ticket what is your expected payoff? Hundred tickets?*

(b) John is looking for a system which guarantees winning this lottery if you buy sufficiently many tickets. Can you help him?

6. *A newsstand owner reckons that the number of buyers of a* RELIABLE GOSSIP *magazine has discrete uniform distribution on [1, 10]. He pays \$ 1 for each copy he orders and sells it for \$ 1.5. How many copies should he order to maximize the expected profit?*

7. *In a Fairpay company 3 employees get 20K per year, 3 get 30K, 2 get 60K, 1 gets 80K and 1 gets 2M. What is the average salary in this company? What is the standard deviation?*

8. *X has discrete uniform distribution on [1, N]. Compute EX and VX.*

9. *During the summer John and Joan are playing 12 matches of tennis. To win a match one needs to win 3 sets. If John wins a set with probability 0.6 and Joan 0.4 what is the probability that John will win at least 8 matches?*

10. *A coin is tossed 100 times. Estimate the probability that head appears between 40 % of the times and 60 % of the times.*

11. *A die is rolled 34 times. What is the most likely number of sixes?*

12. *The probability to win in a lottery is 1/1000. Joe plays 50 times a year for 20 years.*

(a) What is the probability he never wins? wins once? Twice?

(b) If he plays for 40 years find the probability that he wins once during the first 20 years, does not win between 20 and 30 years and wins once between 30 and 40 years.

13. *At a party 100 people bring Christmas cards which are put into a box whereupon each person gets a random card. What is the probability that exactly 3 people get their own cards?*

14. *Joan wants to brake her state jumping record. During the training session she can do it with probability 1/8. During each competition she has three attempts. Let X be the number of competitions before she succeeds. Find E(X).*

15. *Two dice are rolled simultaneously until either the first shows 1 or the second shows an even number. Let X be the number of trials. Find the distribution of X.*

16. *A family decides to have children until they have two boys. Find the probability that they have exactly three girls.*

17. *A box contains 1000 balls, M of which are white and 1000-N are black. n balls are chosen from this box. Compute the probability that exactly three are white if*

(a) $n = 5$, $M = 600$; *(b)* $n = 20$, $M = 50$;