## Joint Distribution.

**1.** *Let $(X, Y)$ have density $p(x, y) = 2y^2 e^{-xy}$ for $x \geq 0$, $0 \leq y \leq 1$ and 0 otherwise. Find marginal distributions of $X$ and $Y$.*

**2.** *Let $(X, Y)$ be uniformly distributed on $[0, 1]^2$. Find the joint distribution of $\max(X, Y)$ and $|X - Y|$.*

**3.** *A box contains 5 red, 10 blue and 15 green balls. 2 balls are chosen at random (a) with replacement; (b) without replacement. Find the joint distribution of the number of red and blue balls as well as their marginal distributions.*

**4.** *Let $X_1, X_2 \dots X_n$ be uniformly distributed on the disc*

$$X_1^2 + X_2^2 + \dots X_n^2 = n.$$

*If $n$ is large, approximate the marginal distribution of $X_1$.*

**5.** *Let $(X, Y)$ be uniformly distributed in the unit circle. Find the probability that*

*(a) $(X, Y)$ is within distance $1/2$ from the origin.*

*(b) $X$ and $Y$ are both positive and $Y/X < 2$.*

**6.** *In a certain state 50% of all cars are American, 30% Japanese and 20% European. 20 cars from that state passed a tall booth.*

*(a) Find a joint distribution of the number of American and number of Japanese cars.*

*(b) Find marginal distribution of the number of American cars.*

*(c) Find the distribution of the number of American cars given that only 2 cars were European.*

**7.** *Let $(X, Y)$ have continuous distribution with smooth density. Suppose that $X$ and $Y$ are independent and also that $X + Y$ and $X - Y$ are independent. Find the distributions of $X, Y$.*

**8.** *Let $X_1, X_2, \dots X_{10}$ be independent uniformly distributed on $[0, 1]$. Find the joint distribution of their maximum and minimum.*

**9.** *Find the distribution of $X + Y$ where $X$ and $Y$ are independent and*

*(a) $p_X(x) = x^{s-1} e^{-x} / \Gamma(s)$, $p_Y(y) = x^{t-1} e^{-x} / \Gamma(t)$.*

*(b) $X \sim \mathcal{N}(1, 2)$, $Y \sim \mathcal{N}(3, 4)$.*

*(c) $X \sim \mathrm{Bin}(10, 1/3)$, $Y \sim \mathrm{Bin}(2, 1/3)$*

*(d) $X \sim \mathcal{P}(1)$, $Y \sim \mathcal{P}(2)$.*

**10.** *Let $X$ and $Y$ be independent having exponential distribution with parameter 1. Find the joint distribution of $X^2$ and $X + Y$.*

**11.** *Let $X_1, X_2, X_3, X_4$ denote the independent random variables variables having exponential distribution with parameter 1. Find the probability that*

$$\max(X_1, X_2, X_3, X_4) - \min(X_1, X_2, X_3, X_4) < 1.$$