

(1) (a) How many ways are there to divide 5 different cakes and 5 identical cookies between 2 people so that the first person gets exactly 3 cakes.

(b) How many ways are there to divide 5 different cakes and 5 identical cookies between 2 people so that the each person gets exactly 5 items.

(2) There are three urns. The first contains 2 red balls and 5 blue balls, the second contains 3 red balls and 4 blue balls and the third contains 4 red balls and 3 blue balls. An urn is chosen at random and then balls are drawn without replacement.

(a) Find the conditional probability that the third urn was chosen given that the first three draws resulted in 2 red and 1 blue ball.

(b) Find the conditional probability that the fourth ball will be blue given that the first three draws resulted in 2 red and 1 blue ball.

(3) (a) 3 people get 3 cards each from the standard 52 card deck. Find the probability that at least one person has only red cards.

(b) 20 tables have 3 palyers and one standard 52 card deck per table. At each table each player is given 3 cards from the deck. Let X be the number of tables where at least one player has only red cards. Compute $P(X = 6)$.

(4) Let X have denisty xe^{-x} if $x \geq 0$ and 0 otherwise.

(a) Compute $P(X > 2)$.

(b) Find the density of $Y = X^2$.

(5) Let (X, Y) have density equal to cxy if $x \geq 0, y \geq 0$ and $x + y \leq 1$ and equal to 0 otherwise.

(a) Compute $E(XY)$.

(b) Compute $P(X > 2Y)$.

(6) Let (X, Y) have density equal to cxy if $x \geq 0, y \geq 0$ and $x + y \leq 1$ and equal to 0 otherwise.

(a) Compute the marginal distribution of X .

(b) Find the moment generating function of X .

(7) Let X and Y be independent, X have uniform distribution on $(0, 1)$ and Y have uniform distribution on $(0, 2)$.

(a) Find the density of $Z = X + Y$.

(b) Find $P(X > Y)$.

(8) N men and N women are divided randomly into N pairs. Let X_N be the number of pairs consisting of two men.

(a) Compute $E(X_N)$ and $V(X_N)$.

(b) Prove a Weak Law of Large Numbers, that is, show that for each ε

$$P\left(\left|\frac{X_N}{N} - \frac{E(X_N)}{N}\right| > \varepsilon\right)$$

if N is large enough.

(9) Let $X_1, X_2 \dots X_{240}$ be independent identically distributed random variables such that X_j has density $3x^2$ if $x \in [0, 1]$ and 0 otherwise. Let $S = X_1 + X_2 + \dots + X_{240}$.

(a) Compute ES and VS .

(b) Find approximately $P(S > 183)$.