(1) (a) We divide the solution into three steps:

(I) Choose committees 1 and 2 ($\binom{30}{3,3,24}$ ways)

(II) Choose comittee 2 ($\binom{30}{3}$ ways)

(III) Choose committee 3 ($\binom{30}{3}$ ways).

Answer: $\binom{30}{3,3,14}\binom{30}{3}^2$.

(b) For each pair (i, j) we have to choose a liason for committees i and j. Alltogather we need to choose six different liasons.

Answer $P_{30,6} = \frac{30!}{24!}$.

(2) (a) We divide the solution into five steps

(I) Choose four classes represented ($\binom{5}{4}$ ways)

(II) Choose the representative for the first class (20 ways)

(III) Choose the representative for the second class (20 ways)

(IV) Choose the representative for the third class (20 ways)

(V) Choose the representative for the fourth class (20 ways)

Answer: $\binom{5}{4} 20^4$

(b) If we forget about the restriction when there are $\binom{100}{4}$ committees.

Among them $5 \times \binom{20}{4}$ one class comiittees (5 ways to choose a class and $\binom{20}{4}$ ways to choose a committee from that class).

Answer: $\binom{100}{4} - 5 \times \binom{20}{4}$

(3) (a) The total number of outcomes is $P_{9,4} = \frac{9!}{5!}$. If 2 is chosen we need to describe

(I) Other three numbers selected from the set of 8 remaining numbers ($\binom{8}{3}$ ways)

(II) In which order the four selected numbers are taken (4! ways).

Answer: $\frac{8!4!}{5!3!}\frac{5!}{9!} = \frac{4}{9}$.

(b) If 2 and 3 chosen we need to describe

(I) Other two numbers selected from the set of 7 remaining numbers ($\binom{7}{2}$ ways)

(II) In which order the four selected numbers are taken (there $4! = 24$ ways but in only half of them 2 comes before 3, so there are 12 possibilities).

Answer: $\frac{7!12}{5!2!}\frac{5!}{9!} = \frac{1}{12}$.

(4) (a) Let $A = \{$Boris next to Ann$\}$, $C = \{$Boris next to Cecilia$\}$. Concentrating on the two neighbours of Boris we see that

$$P(A) = P(C) = \frac{\binom{5}{1}}{\binom{6}{2}} = \frac{1}{3}, \quad P(AC) = \frac{1}{\binom{6}{2}} = \frac{1}{15}.$$

Thus

$$P(A \cup C) = \frac{1}{3} + \frac{1}{3} - \frac{1}{15} = \frac{3}{5}.$$

(b) There are 7! seating arrangment. To describe a good arrangement we need to

(I) Order women (4! ways)

(II) Order men (3! ways)

(III) Choose the seat for the leftmost woman.

Answer: $\frac{4!3!7}{7!} = \frac{1}{5}$.

Alternatively if all women sit together then all men sit together. Let B, D and G denote the events that Boris (respectively Doug or Gary) sit next to two men. As in part (a)

$$P(B) = P(D) = P(G) = \frac{1}{\binom{6}{2}} = \frac{1}{15}$$

so the answer is $P(B) + P(D) + P(G) = \frac{3}{15} = \frac{1}{5}$.

(5) (a) Let $R = \{$red balls are NOT chosen, $B = \{$blue balls are NOT chosen. We have

$$P(R) = \frac{\binom{9}{4}}{\binom{12}{4}}, \quad P(B) = \frac{\binom{8}{4}}{\binom{12}{4}}, \quad P(RB) = \frac{\binom{5}{4}}{\binom{12}{4}}$$

Hence

$$P(R\cup B) = \frac{\binom{9}{4}}{\binom{12}{4}} + \frac{\binom{8}{4}}{\binom{12}{4}} - \frac{\binom{5}{4}}{\binom{12}{4}}$$

Answer:

$$1 - \frac{\binom{9}{4}}{\binom{12}{4}} - \frac{\binom{8}{4}}{\binom{12}{4}} + \frac{\binom{5}{4}}{\binom{12}{4}}.$$

(b) Let $R = \{$red balls are NOT chosen, $B = \{$blue balls are NOT chosen. We have

$$P(R) = \left(\frac{9}{12}\right)^4, \quad P(B) = \left(\frac{8}{12}\right)^4, \quad P(RB) = \left(\frac{5}{12}\right)^4.$$

Hence

$$P(R\cup B) = \left(\frac{9}{12}\right)^4 + \left(\frac{8}{12}\right)^4 - \left(\frac{5}{12}\right)^4.$$

Answer:

$$1 - \left(\frac{9}{12}\right)^4 - \left(\frac{8}{12}\right)^4 + \left(\frac{5}{12}\right)^4.$$

(6) Let V, S, B denote the probability that a random viewer watches volleyball, socccer and Boxing respectively. We have

$$P(VS) = P(V)+P(S)-P(V\cup S) = 0.03, \quad P(VB) = P(V)+P(B)-P(V cupB) = 0.04.$$

Hence

$$P(VSB) = P(V\cup S\cup B)-P(V)-P(S)-P(B)+P(VS)+P(VB)+P(SB) = 0.01.$$

Accoridingly

$$P(BVS^c) = P(VB)-P(VBS) = 0.03, \quad P(BSV^c) = P(BS)-P(BSV) = 0.04$$

and so

$$P(BV^cS^c) = P(B) - P(BVS^c) - P(BSV^c) - P(BVS) = 0.07.$$

(7) (a) If the coin is tossed four times there are $2^4 = 16$ possible outcomes. Thus $P(\text{all heads}) = \frac{1}{16}$, $P(\text{ at least one tail}) = 1 - \frac{1}{16} = \frac{15}{16}$.

(b) Let $A_1 = \{$ tosses 1, 2, 3 and 4 are heads$\}$, $A_2 = \{$ tosses 2, 3, 4 and 5 are heads$\}$, $A_3 = \{$ tosses 3, 4, 5 and 6 are heads$\}$,

Similarly to (a)

$$P(A_1) = P(A_2) = P(A_3) = \frac{1}{2^4} = \frac{1}{16}, \quad P(A_1A_2) = P(A_2A_3) = \frac{1}{2^5} = \frac{1}{32},$$

$$P(A_1A_3) = P(A_1A_2A_3) = \frac{1}{2^6} = \frac{1}{64}.$$

Accordingly

$$P(A_1 \cup A_2 \cup A_3) = \frac{3}{16} - \frac{2}{32} - \frac{1}{64} + \frac{1}{64} = \frac{1}{8}.$$

(8) Let D, H, S, C denote the events that the hand is strong in diamonds (respectively hearts, spades or clubs).

(a) If a hand holds both $A\diamondsuit$ and $K\diamondsuit$ then to describe it we need to specify the remaining 11 cards so

$$P(D) = \frac{\binom{50}{11}}{\binom{52}{13}}.$$

(b) We have

$$P(H) = P(S) = P(C) = P(D) = \frac{\binom{50}{11}}{\binom{52}{13}},$$

$$P(DH) = P(DS) = P(DC)P(HS) = P(HC) = P(SC) = \frac{\binom{48}{9}}{\binom{52}{13}},$$

$$P(HSC) = P(DSC) = P(DHC) = P(DHS) = \frac{\binom{46}{7}}{\binom{52}{13}},$$

$$P(DHSC) = \frac{\binom{44}{5}}{\binom{52}{13}}.$$

Answer

$$4\times\frac{\binom{50}{11}}{\binom{52}{13}}-6\times\frac{\binom{48}{9}}{\binom{52}{13}}+4\times\frac{\binom{46}{7}}{\binom{52}{13}}-\frac{\binom{44}{5}}{\binom{52}{13}}.$$

(9) Let $C1$ be the event that the first coin is chosen and $C2$ be the event that the second coin is chosen.

(a) We have $P(H_1H_2|C1) = (0.5)^2$ $P(H_1H_2|C2) = (0.6)^2$ so by the Law of Total Probability the answer is

$$\frac{1}{2}(0.5)^2+\frac{1}{2}(0.6)^2$$

(b) By Bayes Formula

$$P(C1|H_1H_2)=\frac{\frac{1}{2}(0.5)^2}{\frac{1}{2}(0.5)^2+\frac{1}{2}(0.6)^2}=\frac{25}{61},$$

$$P(C2|H_1H_2)=\frac{\frac{1}{2}(0.6)^2}{\frac{1}{2}(0.5)^2+\frac{1}{2}(0.6)^2}=\frac{36}{61},$$

By the Law of Total Probability

$$P(H|2H1T)=\frac{25}{61}\frac{1}{2}+\frac{36}{61}\frac{6}{10}=\frac{341}{610}\approx 0.559$$

(10) Let I be the event that the patient is ill and P_1, P_2 be the event that test 1, respectively test 2, is positive.

(a) By Bayes Fomula

$$P(I|P_1)=\frac{(0.1)(0.99)}{(0.1)(0.99)+(0.9)(0.1)}\approx 0.524,$$

$$P(I|P_2)=\frac{(0.1)(0.9)}{(0.1)(0.9)+(0.9)(0.03)}\approx 0.769.$$

(b) We have

$$P(P_1P_2|I)=(0.99)(0.9),\quad P(P_1P_2|I^c)=(0.1)(0.03)$$

so by Bayes Formula

$$P(I|P_1P_2)=\frac{(0.1)(0.99)(0.9)}{(0.1)(0.99)(0.9)+(0.9)(0.1)(0.03)}\approx 0.971.$$

(11) (a) Considering three cases depending on which ball is not read we get

$$P(2\text{ reds})=\frac{1}{6}\frac{4}{6}\frac{3}{6}+\frac{5}{6}\frac{2}{6}\frac{3}{6}+\frac{5}{6}\frac{4}{6}\frac{3}{6}=\frac{102}{216}.$$

(b) If there are three color then the last ball must be green. Considering whatever first or second ball are red we get

$$P(X = 3) = \frac{5}{6}\frac{2}{6}\frac{3}{6} + \frac{1}{6}\frac{4}{6}\frac{3}{6} = \frac{42}{216}.$$