STAT410 Midterm 1.

SHOW ALL WORK!!! (No credit will be given for unjustified answers.)

Good luck!

(1) An outdoors club has 100 members. Each member is interested in at least one of the following activities: hiking, biking, kayaking. It is known that 57 people hike, 66 bike, 77 kayak, 30 people hike and bike, 40 people hike and kayak, 50 people bike and kayak.
(a) How many people are interested in only one of the above activity.
(b) Given that a person bikes find the (conditional probability) that they also hike or kayak.

(Remember that a few people enjoy all three activities).

Solution. Let H, B, K denote the events that a person hikes, bikes or kayaks. Then

$$|(H \cup B \cup K)| = |H| + |B| + |K| - |HB| - |HK| - |KB| + |HBK|$$

and so

$$|HBK| = |(H \cup B \cup K)| - |H| - |B| - |K| + |HB| + |HK| + |KB|$$

$$= 100 - 57 - 66 - 77 + 30 + 40 + 50 = 20.$$

Now we can fill Vann diagram for this problem

$$|HBK^c| = 30 - 20 = 10, \quad |HB^cK| = 40 - 20 = 20, \quad |H^cBK| = 50 - 20 = 30,$$

$$|HB^cK^c| = 57 - 10 - 20 - 20 = 7, \quad |H^cBK^c| = 66 - 10 - 20 - 30 = 6,$$

$$|H^cB^cK| = 77 - 20 - 20 - 30 = 7.$$

(a) $|HB^cK^c| + |H^cBK^c| + |H^cB^cK| = 7 + 6 + 7 = 20.$
(b) $P(H \cup K|B) = \frac{P(HBK^c + H^cBK + HBK)}{P(B)} = \frac{60}{66}.$

(2) (a) How many ways are there to distribute 5 red balls numbered from 1 to 5 and 5 blue balls numbered from 1 to 5 between three different boxes.
(b) If one arrangement is chosen at random what is the probability that all red balls are put in the same box?

Solution.
(a) For each ball there are three possibilities. Since there are 10 balls the answer is 3^{10}.
(b) For $j = 2 \dots 5$ let A_j be the event that red ball j goes to the same box as red ball 1. Then $P(A_j) = 1/3$ since and A_j are independent. Thus $P(A_2A_3A_4A_5) = (1/3)^4$.

(3) 5 red, 5 blue and 5 green marbles are divided randomly between three players (named Alice, Bertha and Claudia) so that each player gets 5 marbles.
(a) Find the probability that all Claudia's marbles have the same color.
(b) Find the probability that at least one player has all her marbles of the same color.

(a) There are $\begin{pmatrix} 15 \\ 5 \end{pmatrix}$ ways to choose marbles for Claudia. There are only three there she has all marble of the same color (one just needs to choose the color). Thus thus answer is $\frac{3}{\begin{pmatrix} 15 \\ 5 \end{pmatrix}}$.

(b) Let A, B, C denote the events that Alice, bertha, Claudia have all marbles of the same color. By part (a) $P(A) = P(B) = P(C) = \frac{3}{\begin{pmatrix} 15 \\ 5 \end{pmatrix}}$. Also note that $P(AB) = P(AC) = P(BC) = P(ABC)$. To compute this probability note that there are $\begin{pmatrix} 15 \\ 5,5,5 \end{pmatrix}$ ways to distribute marbles to the players and 3! ways to do so so that each player have the same color (we just need to assign colors to player. Thus

$$P(ABC) = \frac{3!}{\begin{pmatrix} 15 \\ 5,5,5 \end{pmatrix}}.$$

Answer

$$3 \times \frac{3}{\begin{pmatrix} 15 \\ 5 \end{pmatrix}} - 2 \times \frac{3!}{\begin{pmatrix} 15 \\ 5,5,5 \end{pmatrix}}.$$

(4) Patient's symptoms can be caused by one of the three deceases. The doctor estimates that the likelihood that the patient has decease A is 50%, decease B-40%, and decease C-10%. She orders a test which for decease A comes positive 90% of the times, for decease B comes positive 20% of times and for decease C comes positive 50% of times.
(a) Find the probability that the test comes positive.
(b) If the test came positive find the (conditional) probability that the patient has decease A.

Solution. Let P be the event that the test comes positive.
(a) By Law of Total Probability

$$P(P) = P(A)P(P|A) + P(B)P(P|B) + P(A)P(P|C) = 0.58.$$

(b) By Bayes formula

$$P(A|P) = \frac{P(A)P(P|A)}{P(P)} = \frac{0.45}{0.58}.$$