## MATH475 Midterm 2. Due May 4.

## Show all work!!! Good luck!

(1) Which of the graphs on figure 1 are isomorphic (either exhibit an isomorphism or prove that the graphs are not isomorphic)?

(2) Solve the Chinese postman problem for the complete bipartite graph  $K_{3,6}$ .

(3) How many spanning trees of  $K_6$  (the complete graph with vertices 1, 2, 3, 4, 5, 6) are isomorphic to the graph on figure 2?

(4) Find the cheapest spanning tree of the subgraph on figure 3.

(5) Every semester the math department has two teach 2 section each of MATH140, MATH220, and MATH246 and one section each of MATH310, MATH401, MATH410, MATH475. Before the semester starts all professor are submitting their preference lists. Each professor is required to submit at least two course and if the submit a course they agree to teach either one or two sections of this course. The scheduler has two make a schedule so that

- Each professor teaches two sections
- The number of pairs (X, Y) where professor X has to teach course Y but they they do not like it is as small as possible. (If X teaches two sections of Y then the pair (X, Y) is counted twice).
- (a) Convert this problem into a maximal flow network problem.
- (b) Find an optimal schedule for the following preference list

Black: 140, 246, 310 Brown: 140, 220, 246 Green: 140, 401 Orange: 140, 475 White: 140, 310.





Figure 3