STAT 650 Midterm 2.

Show your work!!!

(1) Let X_n be a weakly stationary sequence with spectral density $f(\lambda)$. Find the spectral density of the sequence $Y_n = \frac{X_{n-1}+X_{n+1}}{2}$.
(Remember to normalize the density so that it integrates to 1.)

(2) Consider a regression

$$X_n = \frac{X_{n-1} + X_{n-2} + Y_n}{3}$$

where Y_n is a stationary sequence with zero mean covariance $c(n) = \left(\frac{1}{2}\right)^{|n|}$. Find a distribution of X_0 which makes X_n stationary and compue its covariance function.

(3) Consider Markov chain with states $1, 2, 3$ and the following generator

$$\begin{pmatrix} -1 & 1 & 0 \\ 2 & -6 & 4 \\ 0 & 3 & -3 \end{pmatrix}$$

Compute

$$\lim_{t\to\infty} \frac{1}{t} \int_0^t p_{11}(s)ds.$$

(4) Consider a renewal sequence $T_n = X_1 + X_2 + \ldots X_n$ where X_j are iid having nonlattice distribution and finite expectation. Let $N(t) = \max(n : T_n \leq t)$. Compute

$$\lim_{t\to\infty} P(N(t) \text{ is odd}).$$

Hint. Derive a renewal equation for the above probability in terms of the distribution function F_2 of $X_1 + X_2$.

(5) Consider a delayed renewal sequence $T_n = X_1 + X_2 + \ldots X_n$ where X_j are iid, with X_1 having distribution F^d and X_k having distribution F for $k \geq 2$. Let $N^d(t) = \max(n : T_n \leq t)$, $m^d(t) = E(N^d(t))$. Assume that X_k are nonlattice and that $E(X^2) < \infty$. Compute

$$\lim_{t\to\infty} m^d(t) - \frac{t}{\mu}.$$