

## Midterm 2 sample problems.

**No calculators will be allowed on the midterm, however you can bring one sheet (size A4) with formulas.** Midterm will cover chapters 4, 5 and 6 from the book.

- (1) Three balls are drawn from three urns. The first urn contains 1 blue and 5 red balls, the second urn contains 2 blue and 4 red balls, and the third urn contains 3 red and 3 green balls.
  - (a) Find the probability that 2 red balls are chosen;
  - (b) Let  $X$  be the number of different colors chosen. Find the distribution of  $X$ .
- (2) 10000 bacteria are analyzed in the lab. It is known that the probability that a bacteria has gene A is  $\frac{1}{2}$  and the probability that it has gene B is  $\frac{1}{5000}$ . Compute approximately the probability that
  - (a) 5010 or more bacteria will carry gene A;
  - (b) Exactly 3 bacteria care gene B.
- (3) Jane finds a job which requires her to commute 5 days a week. On her way home Jane is in a hurry so there is  $\frac{1}{20}$  probability that she gets a speeding ticket
  - (a) Let  $X$  be the number of tickets Jane gets during first 6 weeks of work. Compute  $EX$  and  $VX$ .
  - (b) When Jane gets three tickets she needs to attend a driving school. Find the probability that Jane gets her third ticket on her 50th commute.
- (4) A class has 15 boys and 20 girls. 10 theater tickets are distributed at random.
  - (a) Find the probability that girls have exactly 6 tickets;
  - (b) Amanda's lunch mates are Barbara, Cindy, Dalia and Elena. Find the conditional probability that Amanda's table gets exactly 2 tickets given that girls got exactly 6 tickets.
- (5) A number of misprints on a page has Poisson distribution with parameter  $\frac{1}{2}$ .
  - (a) Find the probability that exactly three of the next 10 pages will have at least two misprints.
  - (b) Let  $X$  be the first page which has a misprint. Find  $EX$  and  $VX$ .
- (6) Let  $X_1$  have density equal to  $c_1x^3$  on  $[0, 1]$  and zero elsewhere and  $X_2$  have density equal to  $c_2x^{10}$  on  $[0, 2]$  and zero elsewhere.
  - (a) Compute  $c_1$  and  $c_2$ ;

- (b) Which of the two random variables above has a smaller variance?
- (7) The lifetime of a light bulb (measured in days) has exponential distribution with parameter  $1/100$ .
- (a) Find the distribution of the lifetime measured in hours;
  - (b) If the bulb is installed on a Wednesday at noon, find the probability that it will burn out on a Monday.
- (8) The amount of sales at a department store on a given day has normal distribution with mean 30000 and standard deviation 3000. Find the probability that the store sold
- (a) more than 31000 worth of goods;
  - (b) between 28000 and 32000 worth of goods.
- (9) Let  $X_1$  and  $X_2$  be independent each having density equal to  $2x$  if  $0 \leq x \leq 1$  and equal to 0 otherwise.
- (a) Find  $P(X_1 > 2X_2)$ .
  - (b) Find the distribution of  $X_1 + X_2$ .
- (10) Let  $X_1, X_2, \dots, X_n$  be independent each having density equal to  $2x$  if  $0 \leq x \leq 1$  and equal to 0 otherwise.
- (a) Let  $N$  be the first time  $X_N > \frac{2}{3}$ . Find  $EX$  and  $VX$ .
  - (b) Let  $n = 5$  and let  $X_{(1)} > X_{(2)} > X_{(3)} > X_{(4)} > X_{(5)}$  be the corresponding order statistics. Find  $P(X_{(3)} > X_{(4)} + 0.1)$ .
- (11) Let  $(X, Y)$  have density  $x + y$  if  $0 \leq x \leq 1, 0 \leq y \leq 1$  and equal to zero otherwise.
- (a) Find the marginal distribution of  $X$ .
  - (b) Are  $X$  and  $Y$  independent.
  - (c) Find the distribution of  $Z = X/Y$ .