STAT 650 Midterm 1.

Show your work!!!

(1) A particle on 3×3 squares moves to any of neighboring squares with equal probabailty. (Both horizontal, vertical and diagonal moves are allowed. Thus the state of the chain is one of 9 squares and from a corner there are 3 possible moves, from the center there 8 possible moves and from a side square there are 5 possible moves). Find the stationary distribution.

(2) Consider Markov chain with states $1, 2, 3, 4$ and the following transition matrix

$$\begin{pmatrix} 1/4 & 1/4 & 1/4 & 1/4 \\ 1/2 & 1/4 & 1/4 & 0 \\ 0 & 0 & 1/3 & 2/3 \\ 0 & 0 & 1/2 & 1/2 \end{pmatrix}$$

(a) Classify the states of this chain.
(b) Find λ such that there exists

$$0 < \lim_{n\to\infty} p_{11}(n)/\lambda^n < \infty$$

and compute that limit.
(c) Compute $\lim_{\to\infty} p_{24}(n)$.

(3) Consider two chains on $\{0, 1, 2 \ldots\}$ with transition probabilities p_{ij} and q_{ij} such that $p_{ij} = q_{ij}$ for $i \neq 0$. Show that if both chains are irreducible and one of the chains is recurrent then the other is recurrent.

(4) Consider two irreducible chains on $\{0, 1, 2 \ldots\}$ with transition probabilities p_{ij} and q_{ij}. Consider a new chain whose space is the set of pairs (i, s) where $i \in \{0, 1, 2 \ldots\}$ and $s = 1$ or 2 and transition probabilities

$$r_{(i,1)(j,1)} = p_{ij} \text{ if } i \neq 0, \quad r_{(i,2)(j,2)} = q_{ij} \text{ if } i \neq 0,$$

$$r_{(0,s),(j,1)} = \frac{p_{0j}}{2}, \quad r_{(0,s),(j,2)} = \frac{q_{0j}}{2} \text{ for } s = 1, 2.$$

In other words the state space of the new chain is the union of the state spaces of the two chains except that zero states of both chains are glued together and after coming to zero the particle choose either first or second chain with probability 1/2.

Which of the following statements are true:
(a) If both first and second chain are positively recurrent then the new chain is positively recurrent.

(b) If the first chain is positively recurrent and the second chain is null recurrent then the new chain is null recurrent?

Justify your answers!

(5) Consider the birth chain with intensities $\lambda_n = n+1$. Suppose that $X_0 = 0$ and let T_2 be the time the chain enters state 2. Find the distribution of T_2.

(6) Let $T_1, T_2 \dots T_N$ are points of the Poisson process with intensity 1 on $[0,5]$ (thus N is Poisson(5)). Let $S = \sum_{j=1}^{N} \sin(T_j)$ ($S = 0$ if $N = 0$). Find the expected value and the variance of S.