The exam will consist of four problems devotred to the following for topics

(1) Counting problems including the properties of binomial coefficients (Chapters 2 and 5)
(2) Pigeonhole principle (Chapter 3)
(3) Inclusion-Exclusion formula (Sections 6.1-6.4)
(4) Recurrence Relations (Sections 7.1, 7.4 and 7.5)

All work MUST be shown. No credit will be given for unjustified answers.

Exam will be closed book. However students are allowed to bring one A4 sheet of paper with notes **handwritten by students** on both sides.

Sample problems.

(1) How many ways are there to divide 5 apples and 8 oranges between three people
   (a) without restrictions;
   (b) so that the first person gets exactly 3 oranges;
   (c) so that the first and the second person get the same number of apples?
(2) How many permutations of $\{1, 2 \dots 8\}$ have 1 in the first half and 2 in the second half?
(3) Find $\sum_{k=0}^{n} k \binom{n}{k} 10^k$
(4) Find the smallest k such that if S is a set of k numbers then there is a subset $T \subset S$ such that the sum of numbers in T is divisible by 7?
(5) Let G be the graph whose vertex set consists of three groups X, Y and Z of size 9 each and the vertices are joined by an edge if and only if they belong to different groups. The vertices of G are painted either red or blue. Show that G contains a monochromatic copy of $K_{5,5}$.
(6) How many ways are there to put 3 math, 4 physics and 5 chemistry books on the shelf so that either all math books or all physics books or all chemistry books stay together?
(7) How many permutations of $\{1, 2 \dots 8\}$ have $i_1 \notin \{3, 5\}$ and $i_4 \notin \{1, 3\}$?
(8) Let a_n be the number of strings of length n made of letters A, B and C which have even number of As. Make and solve the recurrence relation for a_n.
(9) Solve the following recurrence relation $a_n = 3a_{n-1} + n2^n$, $a_0 = 1$.