(3) Differentiating the formula $\sum_k \binom{n}{k} x^k = (1+x)^n$ we get $\sum_k k\binom{n}{k} x^{k-1} = n(1+x)^{n-1}$. Multiplying by x we get $\sum_k k\binom{n}{k} x^k = nx(1+x)^{n-1}$. In particular

$$\sum_k k\binom{n}{k} 10^k = 10n \cdot 11^{n-1}$$

(4) Seven numbers is enough. Indeed let $S_k = x_1 + x_2 + \dots + x_k$. Then either $S_1, S_2 \dots S_7$ have different residues mod 7 in which case one of them is divisible by 7 or two number S_k and S_m have the same residue in which case $S_m - S_k$ is divisible by 7. On the other hand 6 numbers may not be enough, e.g.

$S = \{1, 1, 1, 1, 1, 1\}$.

(5) For $U \in \{X, Y, Z\}$ call U blue if it has at least 5 blue vertices and call it red otherwise. Then among X, Y and Z either there are 2 blue or 2 red sets.

(6) Let M, (P, C) denote the events that all math (physics, chemestry) books are together. Then $|M| = 3!\,10!$ (since there are $3!$ ways to order math books and then there are $10!$ to order the set consisting of 9 physics and