MATH 241 EXAM # 1 Sept. 22, 2004

INSTRUCTIONS: Work all problems on the 4 answer sheets provided. Each problem must be on a separate answer sheet. Be sure to put your name, your TA's name and the problem number on each sheet. For problems 1, 2, and 3, show all your work. For all problems, clearly indicate your answers by circling each answer. No programmable or graphing calculators allowed.

1. The position at time t of a particle moving in space is given by the vector

$$\mathbf{r}(t) = e^{2t}\,\mathbf{i} + t\,\mathbf{j} + 2e^{t}\,\mathbf{k}, \quad -1 \le t \le 1$$

a) [10] Find the velocity and acceleration as a function of time.
b) [5] Find the speed as a function of time.
c) [10] Find the tangential and normal components of acceleration a_T and a_N.
d) [10] Find $\mathbf{T}$ and the curvature κ as functions of time.
e) [5] Find the length of the curve parameterized by $\mathbf{r}$.

2. Suppose the curve C_1 is parameterized by $\mathbf{r}_1(t) = t\,\mathbf{i} - t^2\,\mathbf{j} + t^3\,\mathbf{k}$ and the curve C_2 is parameterized by $\mathbf{r}_2(t) = (1+t)\,\mathbf{i} + (t-1)\,\mathbf{j} + \mathbf{k}$.

a) [5] For which curve is the unit normal $\mathbf{N}$ not defined? Why?
b) [10] Find a parametric equation of the line tangent to the curve C_1 at the point $(1, -1, 1)$.
c) [10] Find an equation of the plane tangent to both curves C_1 and C_2 at the point $(1, -1, 1)$.

3.

a) [10] Find the angle between the vectors $\mathbf{i} - 2\mathbf{j} + \mathbf{k}$ and $2\mathbf{i} - \mathbf{k}$. You may leave your answer in terms of inverse trigonometric functions.
b) [10] If $\mathbf{F}$ and $\mathbf{G}$ are differentiable vector valued functions, write down the formulae for the derivatives $(\mathbf{F} + \mathbf{G})'$, $(\mathbf{F} \times \mathbf{G})'$, and $(\mathbf{F} \cdot \mathbf{G})'$.

4. [15] Indicate all correct answers for each question (There may be more than one correct answer or no correct answer.)

i) The dot product of two perpendicular vectors is:
   a) a vector. b) a scalar. c) always zero.
ii) The cross product of two perpendicular vectors is:
   a) a vector. b) a scalar. c) always zero.
iii) If $\mathbf{u}$, $\mathbf{v}$, and $\mathbf{w}$ are vectors parallel to the same plane then $(\mathbf{u} \times \mathbf{v}) \cdot \mathbf{w}$ is:
   a) a vector. b) a scalar. c) always zero.
iv) If a particle travels with constant speed, its velocity $\mathbf{v}(t)$ is:
   a) always zero. b) tangent to the curve it travels.
   c) perpendicular to the curve it travels.
v) If you travel twice as fast around a curve, the curvature will:
   a) increase. b) decrease. c) remain the same.