MATH 241 EXAM \# $2 \quad$ Problem $1 \quad$ Oct. 13, 2004
Name:
TA:
Section:

1. [25] Let $f(x, y, z)=z^{3}+z x-\sin \left(x^{2}-\sqrt{y}\right)$. Consider the surface with equation $f(x, y, z)=0$.
a) Find an equation for the tangent plane to the surface at the point $(-1,1,1)$.
b) Find the largest value of the directional derivative $D_{\mathbf{u}} f$ at the point $(-1,1,1)$.
c) Suppose $z$ is defined implicitly as a function of $x$ and $y$ by $f(x, y, z)=0$. Find $\partial z / \partial x$ when $x=-1, y=1, z=1$.

HONOR PLEDGE: I pledge on my honor that I have not given or received any unauthorized assistance on this examination.

Signature $\qquad$

## MATH 241 EXAM \# $2 \quad$ Problem $2 \quad$ Oct. 13, 2004

Name:
TA:
Section:
2. [25] Find and classify (relative max, relative min, saddle, or degenerate) all the critical points of $f(x, y)=x^{3}-3 x^{2}+y^{5}-20 y+1$. Put your answer on the table below. You may wish to use the blank columns for something useful. If you need more rows or columns you may add them.

| $x$ | $y$ |  | type |
| :--- | :--- | :--- | :--- |
|  |  |  |  |
|  |  |  |  |
|  |  |  |  |
|  |  |  |  |
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|  |  |  |  |

## MATH 241 EXAM \# 2 Problem $3 \quad$ Oct. 13, 2004

Name:
TA:
Section:
3. [25] Find the maximum and minimum of the function $f(x, y)=x^{3}+3 x^{2}+2 y^{2}$ in the elliptical region $2 x^{2}+y^{2} \leq 1$. Put the possible maxima and minima in the table below and indicate which ones are the maxima and which are the minima. If you need more rows or columns you may add them.

| $x$ | $y$ |  |
| :--- | :--- | :--- |
|  |  |  |
|  |  |  |
|  |  |  |
|  |  |  |
|  |  |  |
|  |  |  |

4. [25]
a) Find $\lim _{(x, y) \rightarrow(1,2)}\left(x^{2}-y^{2}\right) /\left(x^{2}+y^{2}\right)$.
b) Find $\lim _{(x, y) \rightarrow(0,0)}\left(x^{3}-y^{2}\right) /\left(x^{2}+y^{2}\right)$.
c) Following are the level curves of four functions. One of them has a relative maximum. Which one? $\qquad$ Which has a saddle point? $\qquad$ Which has a degenerate critical point? $\qquad$ Which has no critical points? $\qquad$ Draw the gradients at the indicated points A, B, C, and D possibly making arbitrary choices.
1) 


2)

3)

4)


