Math 340 Final Exam December 19, 2002

1. (10) Find a basis for the vector space of lower triangular 2×2 matrices. What is the dimension of this vector space?

2. (20) Suppose a matrix A has row echelon form $\begin{pmatrix} 1 & 2 & 0 & 1 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 0 \end{pmatrix}$.

a) What is rank(A)?
b) Find, if possible, a basis for the null space of A.
c) Find, if possible, a basis for the column space of A.

3. (30) Consider the curve C parameterized by $\mathbf{x}(t) = 4\sin t\mathbf{i} + 3\sin t\mathbf{j} + 5\cos t\mathbf{k}$, $0 \le t \le \pi$.
a) Find the tangential and normal components of acceleration a_T and a_N as functions of time.
b) Find the curvature κ when $t = \pi/3$.
c) Find the length of C.
d) Find $\int_C z dx + y dy - dz$

4. (20) Find all points on the surface $x^2 + 3yz = 5$ where the tangent plane is perpendicular to the line through the points $(1, 2, 3)$ and $(2, 1, 0)$. Find an equation of the tangent plane at one of those points.

5. (20) Let S be the portion of the surface $z = 3 - \sqrt{x^2 + y^2}$ in the first octant. Let C be the boundary of S, oriented counterclockwise when viewed from above. Let $\mathbf{F}(x.y, z) = \sin x\mathbf{i} - xz\mathbf{j} + xy\mathbf{k}$.
a) Describe C, (a clear sketch is sufficient).
b) Find $\int_C \mathbf{F} \cdot \mathrm{d}s$.

6. (20) Let D be the solid region inside the cylinder $r = 2$, above the surface $z = x^2 + y^2$, and below the surface $z = 10 + x^2 - 3y^2 + y$. Let S be the boundary of D oriented pointing outward from D. Let $\mathbf{F}(x, y, z) = z\mathbf{i} - x\mathbf{j} + y^2\mathbf{k}$.
a) Find $\int\int_S \mathbf{F} \cdot \mathrm{d}S$.
b) Let S' be obtained from S by deleting the surface $z = x^2 + y^2$, $r \le 2$. Find $\int\int_{S'} \mathbf{F} \cdot \mathrm{d}S$.

7. (20) Let D be the portion of the ellipse $(2x + y)^2 + (x - y)^2 \le 4$ above the line $y = x$. Find $\int\int_D x - y\, dA$. (Hint: use a change of variables $u = 2x + y$, $v = x - y$.)

8. (30) Let A and B be matrices so that $AB = 0$. For each of the following statements, either show the statement is true or give a counterexample to show it is false.
a) Either A or B is zero.
b) The column space of B is a subspace of the null space of A.
c) $\text{rank}A + \text{rank}B \le 7$ if A is a 6×7 matrix.
d) Suppose $g : \mathbb{R}^n \to \mathbb{R}^k$ is differentiable and C is a curve in the level set $g^{-1}(0)$. Let T be a tangent vector to C at a point p of C. Then T is in the null space of $Dg(p)$.

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9. (30) Short answer, true or false. (no justification required). A and B are nonsingular 7×7 matrices.

a) $(AB)^{-1} =$________________.

b) $((A^T - 2B)(I + B))^T =$________________. (Multiply out and simplify as much as possible,)

c) $\det(AB) =$________________.

d) Adding twice the second row to the third row of a 3×4 matrix is the same as multiplying it on the ________________by the matrix ________________.

e) If $\mathbf{v_1}, \ldots, \mathbf{v_p}$ is a linearly independent set of vectors in $\mathbb{R}^m$, then we always have $p \leq m$.

f) If T is a linear transformation, then $T(2\mathbf{v} - \mathbf{w}) = 2T(\mathbf{v}) - T(\mathbf{w})$.

g) Any four vectors which span a four dimensional vector space V form a basis for V.

h) If v_1, v_2, v_3 are linearly independent vectors in a four dimensional vector space V, then there is a vector v_4 so that v_1, v_2, v_3, v_4 is a basis for V.

i) Any two bases of a vector space V have the same number of elements.

j) Every vector space has a finite basis.