Math 340 Exam #1 October 2, 2002

1. (15) Find the volume of the parallelepiped determined by the vectors $(0, 1, 3)$, $(1, 2, -1)$, and $(2, 5, 7)$. Or, if you wish, for extra credit you may find the 4 dimensional volume of the 4 dimensional parallelepiped determined by the vectors $(0, 1, 3, 7)$, $(1, 2, -1, 0)$, $(0, 0, 2, 6)$ and $(2, 5, 7, 1)$ in $\mathbb{R}^4$.

2. (20) Suppose $A$ is a matrix with row echelon form $\begin{pmatrix} 1 & 3 & 4 & 5 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$

a) How many solutions $\mathbf{x}$ are there to $A\mathbf{x} = (1, 2, 3)^T$?

b) Does $A$ have an inverse? If so, what is it, if not, why not?

c) Find all solutions to $A\mathbf{x} = \mathbf{0}$.

3. (20) Let $S \subset \mathbb{R}_{2\times 2}$ be the set of upper triangular matrices.

a) Show that $S$ is a subspace of $\mathbb{R}_{2\times 2}$.

b) Find $v_1, v_2, v_3$ so that $S = \text{Span}\{v_1, v_2, v_3\}$.

4. (20) We'll call a square matrix $P$ a *projection matrix* if $P^2 = P$. Suppose $P$ is a projection matrix and let $Q = I - P$.

a) Show that $Q$ is a projection matrix also.

b) Let $\mathbf{a} = (3, -4)^T \in \mathbb{R}_{2\times 1}$. Find $\text{proj}_{\mathbf{a}}(1, 2)$, the projection of $(1, 2)$ to $\mathbf{a}$.

c) Find a matrix $M$ so that $M\mathbf{b} = \text{proj}_{\mathbf{a}}\mathbf{b}$ for all vectors $\mathbf{b} \in \mathbb{R}_{2\times 1}$.

d) Show that your matrix $M$ is a projection matrix.

5. (25) Short answer. $A$ and $B$ are nonsingular $7 \times 7$ matrices. Answer any five of the following six questions. Be sure to clearly indicate which ones you are answering

a) $(AB)^{-1} =$_______________.

b) $((A^T - 2B)(I + B))^T =$_______________. (Multiply out and simplify as much as possible,)

c) $\det(AB) =$_______________.

d) $\det(A + B) =$_______________.

e) The triangular inequality says _______________.

f) Adding twice the second row to the third row of a $3 \times 4$ matrix is the same as multiplying it on the _______________by the matrix _______________.