Math 340 **Test #2** **Nov. 4, 2002**

Do five of the following six problems. Show enough work to justify your answers

1. (20) Let $f(x, y, z) = x^2 - xe^z - yz$.
 a) Find the gradient of f at the point $(1, 2, 0)$.
 b) Find the largest value of $D_{\mathbf{u}} f$ at the point $(1, 2, 0)$.
 c) Suppose z is defined implicitly as a function of x and y by the formula $f(x, y, z) = 0$. Find $\partial z/\partial x$ when $x = 1$, $y = 2$, $z = 0$.

2. (20) Consider the path parameterized by $\mathbf{x}(t) = 4t\, i + \sin 3t\, j + \cos 3t\, k$.
 a) Find an equation of the line tangent to the path at the point $2\pi\, i - j$.
 b) Find the tangential and normal components of acceleration a_T and a_N as functions of time.
 c) Find $\mathbf{T}$, $\mathbf{N}$ and the curvature κ as functions of time.
 d) Find the length of the path from $t = 0$ to $t = \pi$.

3. (20) Let $F(x, y, z, w) = (4w - x^2 + yz^3)\mathbf{i} + (w^2 z - y^2 - 4x)\mathbf{j}$. Near which of the points $(5, 4, 1, 6)$ or $(1, 0, 4, 1)$ does the implicit function theorem imply that z and w may be written as functions of x and y on the surface $F^{-1}(3, 0)$? For extra credit, you may calculate $\partial z/\partial x$ at one of these points.

4. (20) Let A, B, and C be matrices which all have rank 3. Suppose A is 3×5, B is 5×3, and C is 4×4.
 a) Which of A, B, or C is one to one?
 b) Which of A, B, or C is onto?
 c) Which of A, B, or C is invertible?
 d) Compute the dimensions of the Null Spaces of A, B, and C.

5. (20) Suppose $F\colon \mathbb{R}^n \to \mathbb{R}^k$ and $G\colon \mathbb{R}^k \to \mathbb{R}^n$ and $F \circ G$ is the identity, so $F(G(\mathbf{x})) = \mathbf{x}$ for all $\mathbf{x} \in \mathbb{R}^k$. If $G(\mathbf{x}_0) = \mathbf{y}_0$, show that $DF(\mathbf{y}_0)DG(\mathbf{x}_0) = I_k$. For extra credit, show that if $k = n$ then $G(F(\mathbf{y})) = \mathbf{y}$ for all $\mathbf{y}$ close enough to $\mathbf{y}_0$.

6. (20) Indicate whether each statement is true or false. (no justification required)
 a) If $\mathbf{v_1}, \ldots, \mathbf{v_p}$ is a linearly independent set of vectors in $\mathbb{R}^m$, then we always have $p \geq m$.
 b) If T is a linear transformation, then $T(2\mathbf{v} - \mathbf{w}) = 2T(\mathbf{v}) - \mathbf{w}$.
 c) Any four vectors which span a four dimensional vector space V form a basis for V.
 d) If v_1, v_2, v_3 are linearly independent vectors in a four dimensional vector space V, then there is a vector v_4 so that v_1, v_2, v_3, v_4 is a basis for V.
 e) Any two bases of a vector space V have the same number of elements.
 f) Every vector space has a finite basis.