Math 340 **Test #3** **Dec. 11, 2002**

1. (20) Let D be the region in the first quadrant of $\mathbb{R}^2$ between the hyperbolae $xy = 1$ and $xy = 3$ and the lines $x = y$ and $x = 2y$. Let C be the boundary of the region D, oriented counterclockwise. Let $\mathbf{F}(x, y) = \sin(x^3)\mathbf{i} - x^2y\mathbf{j}$. Find $\int_C \mathbf{F} \cdot \mathrm{d}\mathbf{s}$.

2. (20) Let D be the region in the first octant of $\mathbb{R}^3$ below the paraboloid $z = 4 - x^2 - y^2$ and outside the cylinder $x^2 + y^2 = 1$. Let S be the boundary of D, oriented outwards.
 a) Describe S.
 b) Find the flux integral $\int\int_S \mathbf{F} \cdot \mathbf{n}\, \mathrm{d}S$ where $\mathbf{F}(x, y, z) = x^2\mathbf{i} - (x + y)\mathbf{j} + z\mathbf{k}$.

3. (20) Use Stokes' theorem to find $\int_C \mathbf{F} \cdot \mathrm{d}\mathbf{s}$ where C is the intersection of the elliptic paraboloids $z = x^2 + 3y^2$ and $z = 4 - 3x^2 - y^2$ and $\mathbf{F}(x, y, z) = (x + y)\mathbf{i} - z\mathbf{j} + y\mathbf{k}$, and C is oriented clockwise when viewed from above.

4. (20) Find the area of the surface parameterized by $X(s, t) = (s^2, \sqrt{2}st, t^2)$, $1 \le s \le 2$, $0 \le t \le 1$. Or instead, for an extra 10 points, find the volume of the three dimensional manifold in $\mathbb{R}^4$ parameterized by $X(s, t, u) = (s, t + u, t^2, s)$, $1 \le t \le 2$, $0 \le s \le 2$, $-1 \le u \le 1$.

5. (20) Consider the two vector fields $\mathbf{F}(x, y, z) = e^x\mathbf{i} + 2z^2y\mathbf{j} + (2y^2z - z)\mathbf{k}$ and $\mathbf{G}(x, y, z) = xz\mathbf{i} + x^2y\mathbf{j} + (y - x)\mathbf{k}$.
 a) Find, if possible, functions f and g so $\mathbf{F} = \nabla f$ and $\mathbf{G} = \nabla g$.
 b) Is $\mathbf{F}$ conservative?
 c) Is $\mathbf{G}$ conservative?
 d) Compute $\int_C \mathbf{F} \cdot \mathrm{d}\mathbf{s}$ where C is the line segment running from $(0, 0, 1)$ to $(1, 2, 3)$.
 e) Compute $\int_C \mathbf{G} \cdot \mathrm{d}\mathbf{s}$ where C is the line segment running from $(0, 0, 1)$ to $(1, 2, 3)$.