Math 340 Exam #1 October 6, 2006

1. (20) Which of the following subsets of $\mathbb{R}_{1\times 3}$ are subspaces. Give short reasons for your answers.

a) $\{(x_1, x_2, x_3) \mid x_1 + x_2 = 0\}$.

It is a subspace, it is the null space of $[1\ 1\ 0]$.

b) $\{(x_1, x_2, x_3) \mid x_1 + x_2 + x_3 = 1\}$.

It is not a subspace, it does not contain 0.

c) $\{(x_1, x_2, x_3) \mid e^{x_3} = 1\}$.

It is a subspace since it is the set where $x_3 = 0$.

d) $\{(x_1, x_2, x_3) \mid x_1^2 + x_2^2 + x_3^2 = 1\}$.

It is not a subspace, for example $(1, 0, 0)$ *is in it, but* $2(1, 0, 0)$ *is not.*

2. (20) Let $A = \begin{bmatrix} 1 & 2 & 0 & 3 \\ 0 & 0 & 1 & 5 \\ 2 & 4 & -1 & 1 \end{bmatrix}$.

a) Find a basis for the null space of A.

The row reduced echelon form of A is $\begin{bmatrix} 1 & 2 & 0 & 3 \\ 0 & 0 & 1 & 5 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ *Consequently a basis for the null space of A is* $\left\{ \begin{bmatrix} -2 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -3 \\ 0 \\ -5 \\ 1 \end{bmatrix} \right\}$.

b) Find a basis for the column space of A.

The second column is twice the first, so throw it out. The fourth column is three times the first plus 5 times the third, so throw it out. So the first and third columns are a basis of the column space. You could see this immediately since they are the pivot columns.

c) Find the rank of A.

The rank is 2, the number of pivots.

3. (20) Find the inverse of the partitioned matrix $\begin{bmatrix} A & I_3 \\ B & 0 \end{bmatrix}$ where I_3 is the 3×3 identity, and B is a nonsingular 2×2 matrix. (Hint: try a matrix of the form $\begin{bmatrix} 0 & Y \\ X & Z \end{bmatrix}$.)

$$\begin{bmatrix} A & I_3 \\ B & 0 \end{bmatrix} \begin{bmatrix} 0 & Y \\ X & Z \end{bmatrix} = \begin{bmatrix} X & AY+Z \\ 0 & BY \end{bmatrix}$$ *so* $X = I_3$, $BY = I$ *and* $AY + Z = 0$.

So $Y = B^{-1}$, *and* $Z = -AB^{-1}$. *Thus the inverse is* $\begin{bmatrix} 0 & B^{-1} \\ I_3 & -AB^{-1} \end{bmatrix}$.

4. (20) A linear transformation τ from the vector space $\mathbb{R}[x]$ of polynomials to $\mathbb{R}_{1\times 2}$ satisfies $\tau(x) = (3,4)$, $\tau(x^2-2) = (5,6)$, $\tau(x^3 - x + 1) = (7,8)$, and $\tau(1) = (1,2)$.

a) Find $\tau(x^2 - 2x - 1)$.

$\tau(x^2 - 2x - 1) = \tau(x^2 - 2 + 1 - 2x) = \tau(x^2 - 2) + \tau(1) - 2\tau(x) = (5,6) + (1,2) - 2(3,4) = (0,0)$.

b) Is τ one to one?

τ is not one to one because we saw in part a) that a nonzero vector is mapped to 0.

c) Is τ onto?

τ is onto because $\tau(ax + b) = a(3,4) + b(1,2)$ and $\mathbb{R}_{1\times 2}$ is spanned by $\{(3,4),(1,2)\}$, since they are two linearly independent vectors in a two dimensional space.

5. (20) Short answer. A and B are nonsingular 3×3 matrices.

a) $(AB)^T =$______________.

$B^T A^T$

b) If a vector space V contains a linearly independent set $\{\alpha_1, \alpha_2, \alpha_3, \alpha_4\}$ then $\dim V$ ____4. ($<$, $\le$, $=$, $\ge$, $>$, or $\ne$)

$\ge$

c) If a vector space W is the span of 7 vectors then $\dim W$ ____7. ($<$, $\le$, $=$, $\ge$, $>$, or $\ne$)

$\le$

d) $\det(A^{-1}B^T) =$______________.

$\det B / \det A$

e) Adding twice the first row to the third row of A is the same as multiplying A on the ______________by the matrix ______________.

on the left by $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 2 & 0 & 1 \end{bmatrix}$