

Math 340
 Homework #3
 September 20, 2006

Section 3.1 #1
 3.2 #8

3.1 #1

$$\det A = \det \begin{bmatrix} 5 & 3 \\ -8 & 12 \end{bmatrix} \stackrel{\frac{1}{8}R_2}{=} -8 \det \begin{bmatrix} 5 & 3 \\ 1 & -\frac{3}{2} \end{bmatrix} \stackrel{R_1 - 5R_2}{=} -8 \det \begin{bmatrix} 0 & \frac{9}{2} \\ 1 & -\frac{3}{2} \end{bmatrix}$$

$$\stackrel{R_2 + \frac{3}{2}R_1}{=} -8 \det \begin{bmatrix} 0 & \frac{9}{2} \\ 1 & 0 \end{bmatrix} \stackrel{\frac{2}{9} \cdot R_1}{=} -\frac{21}{9} \cdot 8 \det \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \stackrel{R_1 \leftrightarrow R_2}{=} 84 \det \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \boxed{84}$$

$$\det B = \det \begin{bmatrix} 2 & 3 & 4 \\ 4 & 3 & 1 \\ 1 & 2 & 4 \end{bmatrix} \stackrel{\substack{R_1 - 2R_3 \\ R_2 - 4R_3}}{=} \det \begin{bmatrix} 0 & -1 & -4 \\ 0 & -5 & -15 \\ 1 & 2 & 4 \end{bmatrix} \stackrel{R_1 \leftrightarrow R_3}{=} -\det \begin{bmatrix} 1 & 2 & 4 \\ 0 & -5 & -15 \\ 0 & -1 & -4 \end{bmatrix} \stackrel{R_2 - 5R_3}{=} -\det \begin{bmatrix} 1 & 2 & 4 \\ 0 & 0 & 5 \\ 0 & -1 & -4 \end{bmatrix}$$

$$\stackrel{R_2 \leftrightarrow R_3}{=} \det \begin{bmatrix} 1 & 2 & 4 \\ 0 & -1 & -4 \\ 0 & 0 & 5 \end{bmatrix} \stackrel{\substack{-10R_1; \frac{1}{5}R_3}}{=} -5 \det \begin{bmatrix} 1 & 2 & 4 \\ 0 & -1 & -4 \\ 0 & 0 & 1 \end{bmatrix} \stackrel{R_2 - 4R_3}{=} -5 \det \begin{bmatrix} 1 & 2 & 4 \\ 0 & -1 & -4 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\stackrel{\substack{R_1 - 2R_2; \\ R_1 - 2R_2; R_2 \cdot 4R_3}}{=} -5 \det \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = -5(1) = \boxed{-5}$$

$$\det C = \det \begin{bmatrix} 2 & 1 & -1 & 2 \\ 1 & 3 & 2 & -3 \\ -1 & 2 & 1 & -1 \\ 2 & -3 & -1 & 4 \end{bmatrix} \stackrel{\substack{R_1 - 2R_2 \\ R_3 + R_2 \\ R_4 - 2R_2}}{=} \det \begin{bmatrix} 0 & -5 & -5 & 8 \\ 1 & 3 & 2 & -3 \\ 0 & 5 & 3 & -4 \\ 0 & -9 & -5 & 10 \end{bmatrix} \stackrel{R_1 + R_3}{=} \det \begin{bmatrix} 0 & 0 & -2 & 4 \\ 1 & 3 & 2 & -3 \\ 0 & 5 & 3 & -4 \\ 0 & -9 & -5 & 10 \end{bmatrix}$$

$$\stackrel{-\frac{1}{2}R_1}{=} -2 \det \begin{bmatrix} 0 & 0 & 1 & -2 \\ 1 & 3 & 2 & -3 \\ 0 & 5 & 3 & -4 \\ 0 & -9 & -5 & 10 \end{bmatrix} \stackrel{\substack{R_2 - 3R_1; R_3 - 3R_1; R_4 + 5R_1}}{=} -2 \det \begin{bmatrix} 0 & 0 & 1 & -2 \\ 1 & 3 & 0 & 1 \\ 0 & 5 & 0 & 2 \\ 0 & -9 & 0 & 0 \end{bmatrix} \stackrel{R_1 + R_3}{=} 18 \det \begin{bmatrix} 0 & 0 & 1 & -2 \\ 1 & 3 & 0 & 1 \\ 0 & 5 & 0 & 2 \\ 0 & -9 & 0 & 0 \end{bmatrix}$$

$$\stackrel{R_3 - 5R_1}{=} 18 \det \begin{bmatrix} 0 & 0 & 1 & -2 \\ 1 & 3 & 0 & 1 \\ 0 & 0 & 0 & 2 \\ 0 & -9 & 0 & 0 \end{bmatrix} \stackrel{\frac{1}{2}R_3}{=} 36 \det \begin{bmatrix} 0 & 0 & 1 & -2 \\ 1 & 3 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & -9 & 0 & 0 \end{bmatrix} \stackrel{R_1 + 2R_3; R_2 + R_3}{=} 36 \det \begin{bmatrix} 0 & 0 & 1 & 0 \\ 1 & 3 & 0 & 2 \\ 0 & 0 & 0 & 1 \\ 0 & -9 & 0 & 0 \end{bmatrix}$$

$$\stackrel{R_1 \leftrightarrow R_2}{=} -36 \det \begin{bmatrix} 1 & 3 & 0 & 2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & -9 & 0 & 0 \end{bmatrix} \stackrel{R_1 - 3R_2}{=} -36 \det \begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & -9 & 0 & 0 \end{bmatrix} \stackrel{R_1 - 2R_3}{=} -36 \det \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & -9 & 0 & 0 \end{bmatrix} = -36(1) = \boxed{-36}$$