If A and C commute then $AC = CA$

$$\begin{aligned}\det(A)\det(D-CA^{-1}B) &= \det(A(D-CA^{-1}B)) \\ &= \det(AD - ACA^{-1}B) \\ &= \det(AD - CAA^{-1}B) \\ &= \det(AD - CB).\end{aligned}$$

A and B commute then $AB = BA$

$$\begin{aligned}\det(A)\det(D-CA^{-1}B) &= \det(D-CA^{-1}B)\det(A) \\ &= \det((D-CA^{-1}B)A) \\ &= \det(DA - CA^{-1}BA) \\ &= \det(DA - CA^{-1}AB) \\ &= \det(DA - CB).\end{aligned}$$

Note: $\det(A-B) \neq \det(A) - \det(B)$

Counter example:

$$\begin{array}{c} A \\ \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \end{array} - \begin{array}{c} B \\ \begin{bmatrix} 3 & 2 \\ 1 & 4 \end{bmatrix} \end{array} = \begin{bmatrix} -2 & 0 \\ 2 & 0 \end{bmatrix}$$

$\det(A) = 4 - 6 = -2$

$\det(B) = 12 - 2 = 10$

$\det(A-B) = 0 - 0 = 0.$

$\det A - \det B = -12 \neq \det(A-B)$!