Math 341 Exam #1 March 7, 2003

1. (15) Let $A = \begin{pmatrix} 2 & 1 & -1 \\ 0 & 0 & 2 \\ 0 & 0 & 2 \end{pmatrix}$

a) Find all eigenvalues and eigenvectors of A. Some calculators can do this for you, so show enough work to let me know you can do it by hand.

b) If possible, find a matrix P so $P^{-1}AP$ is diagonal. If this is not possible, say why not.

2. (15) Let S be the subspace of $\mathbb{R}^4$ spanned by $(1,1,1,1)^T$, $(1,0,3,0)^T$, and $(1,-1,0,0)^T$. Find an orthogonal basis for S.

3. (20) Find and classify (local mx, local min, saddle, or degenerate) all critical points of $f(x,y,z) = 2x^3 - 3x^2 + y^5 - 20y + y^3z^2$.

4. (20) Use Lagrange multipliers to do one of the following:

a) Find the points on the surface $xy^2 + 4z^2 = 16$ which are closest and furthest from the origin.

b) Let a and b be perpendicular unit vectors in $\mathbb{R}^n$. Let T be the set of points x on the sphere $||x|| = \sqrt{2}$ where $a \cdot x = 1$. Find the maximum and minimum of $b \cdot x$ for $x \in T$.

5. (15) Prove one of the following:

a) Suppose B is orthogonally diagonalizable, that is, there is an orthogonal matrix Q so that $Q^{-1}BQ$ is diagonal. Show that B is symmetric.

b) Recall a square matrix P is a projection matrix if $P^2 = P$. Show that a projection matrix Phas at most two different eigenvalues. (Hint, take an eigenvector v and write Pv in two ways.)

6. (15) Prove one of the following:

a) If A is any $k \times n$ matrix then all eigenvalues of A^TA are real and nonnegative. (Hint: If v is an eigenvector of A^TA, consider the dot product of v and A^TAv. Write this as a product of matrices and simplify in two ways.)

b) If C is any square matrix show that C and C^T have the same eigenvalues. (Hint, compare their characteristic polynomials). Also, prove or disprove whether or not they have the same eigenvectors.