Math 341 Exam #2 April 14, 2003

1. (15) Find all solutions of $y\cos(yt) + e^t + t\cos(yt)y' = 0$.

2. (25) Find all solutions of $y''' - 3y'' + 3y' - y = e^t$. Or, for a 10 point penalty, you may find all solutions of $y'' - 2y' + y = e^t$.

3. (10) Find the Laplace transform of $f(t) = \begin{cases} t^2 & \text{if } t < 1 \\ 1 & \text{if } 1 \le t \le 3 \\ \sin t & \text{if } t > 3 \end{cases}$.

4. (20) Solve $y'' + 2y' = \delta(t-2)$, $y(0) = 1$, $y'(0) = 2$.

5. (18) What does the existence and uniqueness theorem say about each of the following IVPs:

a) $ty'' + y' - y^2 = t$, $y(1) = 2$, $y'(1) = 4$.

b) $ty'' + y' - y^2 = t$, $y(0) = 2$, $y'(0) = 4$.

c) $ty'' + (\sin t)y' - (e^t - 1)y^2 = t$, $y(0) = 2$, $y'(0) = 4$.

6. (12) Suppose y_1 is a solution to $ty''' + e^t y' - y = 1$ and y_2 is a solution to $ty''' + e^t y' - y = e^t$ and y_3 is a solution to $ty''' + e^t y' - y = 3$. Which of the following statements must be true?

a) $y_2 = e^t y_1$.

b) $y_3 = 3y_1$.

c) $3y_1 - y_2$ is a solution to $ty''' + e^t y' - y = 3 - e^t$.

d) $3y_1 - y_3$ is a solution to $ty''' + e^t y' - y = 0$.