Math 341 Final Exam May 15, 2007

1. (15) Solve $y' + ty = t$, $y(1) = 2$.

2. (45) A symmetric matrix A has characteristic polynomial $(\lambda-1)^2(\lambda-3)$ and $A\begin{pmatrix}1\\2\\2\end{pmatrix} = \begin{pmatrix}1\\2\\2\end{pmatrix}$, and $A\begin{pmatrix}0\\1\\1\end{pmatrix} = \begin{pmatrix}0\\1\\1\end{pmatrix}$.

a) Find, if possible, an orthogonal matrix P so that $P^{-1}AP$ is diagonal. If this is not possible, say why not.

b) Classify the critical point 0 of the function $f(x) = x^T Ax$ (local maximum, local minimum, saddle, or degenerate).

c) Solve the differential equation $x' = Ax$, $x(0) = \begin{pmatrix}1\\1\\-1\end{pmatrix}$.

3. (35) Is the region $x^2 + y^2 \leq 4$ compact? Find the maximum and minimum of $f(x, y) = x^2y + 2x^2 + 2(y + 1)^2$ in the region $x^2 + y^2 \leq 4$.

4. (20) Note that $y = 1/x$ solves $xy'' + 2(1 - x)y' - 2y = 0$.

a) Find another linearly independent solution to $xy''+2(1-x)y'-2y = 0$.

b) Find all solutions to $xy'' + 2(1 - x)y' - 2y = 6$.

5. (30) Find all the equilibrium points of the system $x' = y^2-xy+2y-2xy^2$, $y' = x - y$. Sketch the orbits near each equilibrium point. For full credit your sketches should account for the eigenvectors. Determine the stability near each equilibrium point.

6. (15) Find the Laplace transform $\mathcal{L}(y) = Y(s)$ of the solution of the initial value problem $y'' + 2y' + y = \pi te^{-t} + 2\delta(t - \pi)$, $y(0) = 1$, $y'(0) = 2$. Do not solve for y, just find $\mathcal{L}(y)$.

7. (15) Find the inverse Laplace transform $\mathcal{L}^{-1}(e^{-3\pi s}/(s^2 - 2s + 2))$.

8. (25) Find the general solution of the differential equation $y''+2y'-3y = \cos(2t) + 12e^t$, $y(0) = 1$, $y'(0) = 0$.