

1. [20] Find and classify all critical points of the function $f(x, y, z) = x^2 + 5y^2 + 4xy + 3z^2 - z^3$.

2. [20] Use Lagrange multipliers to find the critical points of $f(x, y, z) = x + 2y + 3z$ subject to the constraints $z^2 - y^2 = 3$ and $y + 2x = 1$.

3. [20] Find an orthonormal basis of \mathbb{R}^3 by applying the Gram-Schmidt process to the basis $\begin{bmatrix} 2 \\ 2 \\ -1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$.

4. [20] Find all characteristic values and characteristic vectors of $A = \begin{bmatrix} 0 & 1 & 2 \\ 0 & 2 & 6 \\ 0 & 0 & 0 \end{bmatrix}$. Determine whether or not A is similar to a diagonal matrix.

5. [20] Suppose B is a singular symmetric 3×3 matrix, $B \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \\ 2 \end{bmatrix}$, and $B \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 2 \\ 2 \end{bmatrix}$.

a) What are the characteristic values and characteristic vectors of B ?

b) Find an orthogonal matrix P so that $P^T B P$ is diagonal.