Math 405 Diagnostic Test
Due Wednesday, September 6

1. Tell me the conditions which applied when you took this test. Open or closed book? If open book, what book or books? How long did it take you? Did you use a calculator? A computer? If so, what computer program? Any other relevant info?

2. Let A be the matrix

$$A = \begin{bmatrix} 1 & 2 & 2 & 0 & 1 \\ 2 & 4 & 4 & 2 & 4 \\ 2 & 3 & 4 & 0 & 0 \\ 1 & 3 & 2 & 2 & 5 \end{bmatrix}$$

a) Find the reduced row echelon form of A.
b) Find the rank of A.
c) Find a basis for the column space of A.
d) Find a basis for the Null space of A.
e) Find all solutions to $A\mathbf{x} = [1 \quad 0 \quad 1 \quad 0]^T$.
f) Find all solutions to $A\mathbf{x} = [2 \quad 2 \quad 1 \quad 0]^T$.

3. For each of the following, give an example of such a matrix in reduced row echelon form. If it is not possible to have such a matrix in reduced row echelon form, say why it is not.
a) A 3×4 matrix whose columns do not span $\mathbb{R}^3$.
b) A 3×4 matrix A so that $A\mathbf{x} = \mathbf{b}$ has a solution for every $\mathbf{b}$ in $\mathbb{R}^3$.
c) A 4×3 matrix A so that $A\mathbf{x} = \mathbf{0}$ has more than one solution.
d) A 3×3 matrix A so that $A\mathbf{x} = [0\ 0\ 1]^T$ has more than one solution.
e) A 4×3 matrix A whose columns are linearly dependent.
f) A 4×3 matrix A so that $A\mathbf{x} = \mathbf{0}$ has no solutions.

4. Determine whether each of the following subsets of $\mathbb{R}^5$ are subspaces and find a basis and dimension if they are.
a) S_1 is the set of $[x_1, x_2, x_3, x_4, x_5]^T$ so that $x_1 + x_2 + x_3 + x_4 + x_5 = 1$.
b) S_2 is the set of $[x_1, x_2, x_3, x_4, x_5]^T$ so that $x_1 + x_2 + x_3 + x_4 + x_5 = 0$ and $x_1 + x_2 + x_3 = x_4 + x_5$.
c) S_3 is $\text{Span}\{[1, 2, 3, 4, 5]^T, [1, 1, 1, 1, 1]^T, [0, 1, 2, 3, 4]^T\}$.

5. Indicate whether each statement is true or false.
a) If A and B are square matrices of the same size then $AB = BA$.
b) If A and B are square matrices of the same size then $A + B = B + A$.
c) If A and B are invertible and the same size, then AB is invertible and $(AB)^{-1} = A^{-1}B^{-1}$.
d) If $\{\mathbf{v_1}, \mathbf{v_2}, \mathbf{v_3}, \mathbf{v_4}\}$ is linearly dependent then $\mathbf{v_4}$ is always a linear combination of $\mathbf{v_1}, \mathbf{v_2}$, and $\mathbf{v_3}$.
e) If $\{\mathbf{v_1}, \mathbf{v_2}, \mathbf{v_3}, \mathbf{v_4}\}$ is a set of vectors in $\mathbb{R}^5$ and $\{\mathbf{v_1}, \mathbf{v_2}, \mathbf{v_3}, \mathbf{v_4}\}$ is linearly independent then $\{\mathbf{v_1}, \mathbf{v_2}, \mathbf{v_3}\}$ is always linearly independent.