Matlab in Math 461, part five

Eigenvalues and Eigenvectors, etc.

If A is a square $n \times n$ matrix, then the command `E = eig(A)` will produce a vector E whose entries are the n eigenvalues of A (including multiplicities). Some eigenvalues may be complex and the real and imaginary parts will be given. To find the eigenvectors, use the command

```
>> [V D] = eig(A)
```

Then D will be a diagonal matrix whose diagonal entries are the eigenvalues (so D = diag(E)) and V will be an $n \times n$ matrix whose columns are the corresponding eigenvectors. In particular, you are guaranteed that $AV = VD$.

If A has n distinct eigenvalues, then we know $\mathbb{R}^n$ has a basis of eigenvectors, so V will be invertible. If there are repeated eigenvalues, however, it may happen that there is not a basis of eigenvectors so V will not be invertible.

Other matlab operations

The trace of a matrix is the sum of its diagonal entries. In matlab just write `trace(A)`. To find the characteristic polynomial of a matrix A, just type `poly(A)` which will return a vector of coefficients of the characteristic polynomial. To find the roots of this polynomial (which are of course the eigenvalues of A) use the matlab command `roots`. The command `roots(poly(A))` is less accurate and more time consuming than the equivalent `eig(A)`. In fact, matlab calculates poly(A) not by evaluating the determinent of $\lambda I - A$ but by first finding eig(A) and then multiplying out the polynomial which has those eigenvalues for roots! Note: the characteristic polynomial calculated by matlab differs slightly from that in the book; it is $\det(\lambda I - A)$ versus $\det(A - \lambda I)$. This results in a sign difference for odd sized matrices.

Problems due April 8

Problem 1: Generate some random square matrices and calculate the following quantities: the sum of the eigenvalues, the product of the eigenvalues, the characteristic polynomial, the trace and the determinent. Formulate conjectures on any interrelationships between these things. (Recall the matlab commands sum and prod give the sum and product of a vector's entries).

Problem 2: sec 5.1 prob 37

Problem 3: sec 5.2 prob 28

Problem 4: For the matrix of sec 5.3 prob 33 find P and a diagonal D so this matrix is $P^{-1}DP$.

Problem 5: sec 5.4 prob 32