Math 461 Exam #2 Nov. 17, 2000

1. (30) Let $S = Span(\ (1,0,1,0)^T, (0,1,0,1)^T, (1,2,3,4)^T\ )$.
 a) Find an orthogonal basis for S.
 b) Find a basis for $S^\perp$.

2. (20) Let u_1, u_2, u_3 be an orthonormal set in an inner product space V.
 a) Calculate $||u_1 - 2u_2 + 4u_3||$.
 b) Show that $u_1 - 2u_2$ is orthogonal to $2u_1 + u_2$.
 c) Are u_1, u_2, u_3 linearly independent?
 d) Is u_1, u_2, u_3 a basis for V?

3. (25) Let $L\colon P_3 \to P_4$ be the map $L(p) = xp(x)$.
 a) Show that L is a linear transformation.
 b) Find the Kernel and Range of L.
 c) Find the matrix of L with respect to the bases $[x-1,\ 1,\ x^2]$ of P_3 and $[1,\ x,\ x^2,\ x^3]$ of P_4.

4. (20) Show that if A is an $m \times n$ matrix and x is in $N(A^TA)$, then x is in $N(A)$.

5. (30) True (always true), False (always false), Maybe (sometimes true and sometimes false, depending on A, S, etc.) or short answer. A is a 4×8 matrix, and S is a subspace of a seven dimensional inner product space V. Also A has rank 3 and S has dimension 3.

 a) The null space $N(A)$ has dimension 1.
 b) If $x \perp y$ and $y \perp z$ then $x \perp z$.
 c) Let x_1 and x_2 be two different least squares solutions to $Ax = b$. Then $Ax_1 = Ax_2$.
 d) If $[u_1, u_2, u_3]$ is a basis for S, and $x \perp u_i$ for each i, then x is in $S^\perp$.
 e) If $AB = 0$ then the column space of B is contained in $N(A)$.
 f) If B and C are similar matrices then $\det(B) = \det(C)$.
 g) $\dim S^\perp =$ ___.
 h) $\dim N(A^T) =$ ___.
 i) Give an example of an inner product on $C[0,2]$.
 j) If $[u_1, u_2, u_3]$ is an orthogonal basis for S, what is the formula for the projection of x to S?