Math 461 Exam #2 April 15, 1993

1. (30) Let $A = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 2 & 4 \\ -3 & 2 & -1 \\ 0 & 2 & 5 \end{pmatrix}$. Find an orthonormal basis for the column space of A. Find the QR factorization of A. Find the least squares solution to $Ax = (1 \quad 2 \quad 3 \quad 4)^T$.

2. (20) Let $A = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$. Find the matrix of the linear transformation L_A with respect to the basis $\left[\begin{pmatrix} 2 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \end{pmatrix} \right]$ of R^2.

3. (20) Let u_1, u_2, u_3 be an orthonormal set in an inner product space V.

a) Calculate $||u_1 - 2u_2 + 4u_3||$.

b) Show that $u_1 - 2u_2$ is orthogonal to $2u_1 + u_2$.

c) Are u_1, u_2, u_3 linearly independent?

d) Is u_1, u_2, u_3 a basis for V?

4. (15) What are similar matrices? Give an example of two different 2×2 matrices which are similar. Give an example of two 2×2 matrices which are not similar and say why they are not.

5. (15) Let A and B be matrices so that $AB = 0$. Show that the column space of A^T is orthogonal to the column space of B.

6. (25) Let V be an inner product space and let x be any vector in V. Define a map $L_x : V \to R$ by setting $L_x(v) = \langle x, v \rangle$ for all $v \in V$.

a) Show that L_x is a linear transformation.

b) Find the Kernel and Range of L_x if x is not 0.

c) Find the Kernel and Range of L_0.

d) Show that if V is finite dimensional and $K: V \to R$ is any linear transformation, then $K = L_x$ for an appropriate x. (Hint: Consider an orthonormal basis of V and the matrix of K with respect to this basis.)