Math 461 Mini Test # 2, section 0601

Instructions: Do three of the following four problems. Choose one of the three to be worth ten points instead of 20.

Name:__________________

20 point problems: _________and _________

10 point problem: _________

1. (20) Consider the following matlab output.

```
>> A =  [3 6 1 1;1 2 3 -5; 2 4 0 2]
A =
       3     6     1     1
       1     2     3    -5
       2     4     0     2
>> rref(A)
ans =
       1     2     0     1
       0     0     1    -2
       0     0     0     0
```

a) What is the rank of A?

The rank is 2 since there are two pivots.

b) Find bases for the column space of A, the row space of A, and the null space of A and determine the dimensions of these vector spaces.

A basis for the row space is $\{[1\ 2\ 0\ 1]^T, [0\ 0\ 1\ \ -2]^T\}$. *A basis for the column space is the pivot columns* $\{[3\ \ 1\ \ 2]^T, [1\ \ 3\ \ 0]^T\}$. *From the rref, a basis for the null space is* $\{[-2\ 1\ 0\ 0]^T, [-1\ 0\ 2\ 1]^T\}$. *All have dimension two.*

2. (20) Let H be the set of 2×2 symmetric matrices. (A matrix A is symmetric if $A = A^T$. So for example I_2, $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$, and $\begin{bmatrix} 2 & 3 \\ 3 & 1 \end{bmatrix}$ are all symmetric. A 2×2 matrix is symmetric if and only if its two off-diagonal entries are equal.)

a) Show that H is a subspace of $M_{2\times 2}$.

The zero matrix is symmetric. If A *and* B *are symmetric, then* $(A+B)^T = A^T + B^T = A + B$ *so* $A + B$ *is symmetric. If* A *is symmetric and* c *is a scalar then* $(cA)^T = cA^T = cA$ *so* cA *is symmetric. So the symmetric matrices form a subspace.*

b) Find a basis for H.

Symmetric matrices have the form $\begin{bmatrix} a & c \\ c & b \end{bmatrix}$ *so one example of a basis is given by* $\{\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}\}$. *Those three matrices are clearly linearly independent and span the symmetric matrices.*

c) What is the dimension of H?

The dimension is three since a basis has three elements.

3. (20) Let $\mathcal{B} = \{1, t-1, t^2-2t+1\}$ and $\mathcal{C} = \{t, 1, t^2\}$ be two bases for P_2. Find the coordinates $[t^2+3t-4]_{\mathcal{B}}$ and $[t^2+3t-4]_{\mathcal{C}}$. In other words, find the $\mathcal{B}$-coordinate vector for t^2+3t-4 and find the $\mathcal{C}$-coordinate vector for t^2+3t-4.

By inspection, $[t^2+3t-4]_{\mathcal{C}} = [3 \;\; -4 \;\; 1]^T$. *We could find*

$$[t^2+3t-4]_{\mathcal{B}} = P_{\mathcal{B}\leftarrow\mathcal{C}} \begin{bmatrix} 3 \\ -4 \\ 1 \end{bmatrix} = P^{-1}_{\mathcal{C}\leftarrow\mathcal{B}} \begin{bmatrix} 3 \\ -4 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 & 1 & -2 \\ 1 & -1 & 1 \\ 0 & 0 & 1 \end{bmatrix}^{-1} \begin{bmatrix} 3 \\ -4 \\ 1 \end{bmatrix}$$

which could be solved by row reducing $\begin{bmatrix} 0 & 1 & -2 & 3 \\ 1 & -1 & 1 & -4 \\ 0 & 0 & 1 & 1 \end{bmatrix}$. *You could also see by inspection that* $t^2 = (t^2-2t+1) + 2(t-1) + 1$ *and* $t = (t-1)+1$ *so*

$$t^2+3t-4 = (t^2-2t+1)+2(t-1)+1+3(t-1)+3\cdot 1-4\cdot 1 = (t^2-2t+1)+5(t-1)+0\cdot 1$$

So $[t^2+3t-4]_{\mathcal{B}} = [0 \;\; 5 \;\; 1]^T$.

4. (20) Short answer:

a) $\{b_1, b_2, \ldots, b_n\}$ is a basis for a subspace H of V if _________.

it is linearly independent and spans H.

b) If $\{b_1, b_2, \ldots, b_n\}$ and $\{c_1, c_2, \ldots, c_k\}$ are two bases for a vector space V then what is the relation between n and k?

n=k

c) True or false: Every vector space has a basis.

False. For example $C[0,1]$ *has no basis since it is infinite dimensional. This would be true if you allowed infinite bases, but in this undergraduate course we don't.*

d) If A is an $m \times n$ matrix, then $\text{rank}(A) + \dim Nul(A) =$_________.

n

e) If $\{v_1, v_2, v_3, v_4\}$ is a linearly independent set in a finite dimensional vector space V then the dimension of V is $= 4$, > 4, < 4, ≥ 4, ≤ 4, or none of the above?

≥ 4