Math 461 Test # 1

Write your name here:

1. (25) let $A = \begin{bmatrix} 1 & 2 & 3 \\ -1 & 1 & 0 \\ 2 & 7 & 9 \end{bmatrix}$ and $\mathbf{b} = \begin{bmatrix} 14 \\ 1 \\ 53 \end{bmatrix}$.

   a) Find all solutions to $Ax = \mathbf{b}$. Do this by writing down the augmented matrix and performing row operations. You must show your work.

   b) Is $\mathbf{b}$ a linear combination of the columns of A? If not, say why not. If it is, find the weights (coefficients) of a linear combination.

2. (20) Short answer.

   a) How many vectors are in the span of $\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ and $\begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix}$?

   b) Let A be a 7×5 matrix. How many pivot columns must A have so that its columns will be linearly independent?

   c) Let A be the 3×3 zero matrix (all its entries are 0). Find all solutions to $A\mathbf{x} = \mathbf{0}$.

   d) Let A be the zero matrix in c) above. What is the span of the columns of A?

3. (27) Suppose you have a matrix A and after doing some row operations you obtain one of the four matrices below. Answer the following questions about the matrix A. For example, the correct answer to i) is matrices a and c, since doing row operations does not change the number of columns of a matrix.

a) $\begin{bmatrix} 1 & 3 & 0 & -2 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$ b) $\begin{bmatrix} 1 & 3 & 0 \\ 0 & 1 & -1 \\ 0 & 1 & -2 \\ 0 & 0 & 1 \end{bmatrix}$

c) $\begin{bmatrix} 1 & 3 & 0 & -2 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 2 \end{bmatrix}$ d) $\begin{bmatrix} 2 & 3 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 3 \end{bmatrix}$

   i) In which case(s) did A have five columns?

   ii) In which case(s) do the columns of A span $\mathbb{R}^4$.

   iii) In which case(s) do the columns of A span $\mathbb{R}^3$.

   iv) In which case(s) did you put A in reduced echelon form.?

   v) In which case(s) does the equation $A\mathbf{x} = \mathbf{b}$ have a solution for any b (with the right number of rows)?

   vi) In which case(s) does the equation $A\mathbf{x} = \mathbf{0}$ have infinitely many solutions?

   vii) In which case(s) does the equation $A\mathbf{x} = \mathbf{0}$ have no solutions?

   viii) In which case(s) is the linear transformation $T(\mathbf{x}) = A\mathbf{x}$ one to one?

   ix) In which case(s) is the linear transformation $T(\mathbf{x}) = A\mathbf{x}$ onto?

see next page for the rest of the problems

4. (20) Let A be the matrix $\begin{bmatrix} 1 & 2 \\ -1 & 1 \\ -2 & 4 \end{bmatrix}$.

a) Write $\begin{bmatrix} 2 \\ -5 \\ -12 \end{bmatrix}$ as a linear combination of the columns of A.

b) Is $\begin{bmatrix} 2 \\ -5 \\ -12 \end{bmatrix}$ in the span of the columns of A?

c) Let T be a linear transformation so that $T(\begin{bmatrix} 1 \\ -1 \\ -2 \end{bmatrix}) = \begin{bmatrix} 7 \\ 6 \end{bmatrix}$ and $T(\begin{bmatrix} 2 \\ 1 \\ 4 \end{bmatrix}) = \begin{bmatrix} 3 \\ 4 \end{bmatrix}$.

What is $T(\begin{bmatrix} 2 \\ -5 \\ -12 \end{bmatrix})$?

5. (33) Indicate whether each statement is true or false.

a) The span of two vectors is always a plane.

b) If $\mathbf{v_1}, \ldots, \mathbf{v_p}$ is a list of vectors in $\mathbb{R}^m$ and $p > m$ then this list is always linearly dependent.

c) If the span of $\mathbf{v_1}, \ldots, \mathbf{v_p}$ is $\mathbb{R}^m$, then we always have $p \geq m$.

d) A vector $\mathbf{b}$ is a linear combination of the columns of a matrix A if and only if the equation $A\mathbf{x} = \mathbf{b}$ has at least one solution.

e) The equation $A\mathbf{x} = \mathbf{b}$ is always consistent if the augmented matrix $[A \quad \mathbf{b}]$ has a pivot position in every row.

f) If T is a linear transformation, then $T(\mathbf{0}) = \mathbf{0}$.

g) If the columns of an $m \times n$ matrix A span $\mathbb{R}^m$, then the equation $A\mathbf{x} = \mathbf{b}$ is consistent for each $\mathbf{b}$ in $\mathbb{R}^m$.

h) A homogeneous equation is always consistent.

i) If $\mathbf{v_1}, \mathbf{v_2}, \mathbf{v_3}, \mathbf{v_4}$ is a linearly dependent list of vectors then $\mathbf{v_4}$ is always a linear combination of $\mathbf{v_1}, \mathbf{v_2}$, and $\mathbf{v_3}$.

j) If $\mathbf{v_1}, \mathbf{v_2}, \mathbf{v_3}, \mathbf{v_4}$ are in $\mathbb{R}^6$ and $\mathbf{v_1}, \mathbf{v_2}, \mathbf{v_3}$ are linearly dependent then $\mathbf{v_1}, \mathbf{v_2}, \mathbf{v_3}, \mathbf{v_4}$ are always linearly dependent.

k) If $\mathbf{v_1}, \mathbf{v_2}, \mathbf{v_3}, \mathbf{v_4}$ are linearly independent vectors in $\mathbb{R}^6$ then $\mathbf{v_1}, \mathbf{v_2}, \mathbf{v_3}$ are always linearly independent.