Math 461 Test # 2

1. (25) For each set H below determine whether or not it is a subspace. If it is not a subspace, show why it is not. If it is a subspace and is finite dimensional, write down a basis for H and determine the dimension of H.

a) H is the set of upper triangular matrices in $\mathbb{M}_{3\times 3}$.

b) $H = Span\{v_1, v_2, v_3\}$ where v_i are nonzero vectors in a vector space V, $v_1 = 2v_2 - 5v_3$, and v_2 is not a scalar multiple of v_3.

c) H is the set of polynomials p in $\mathbb{P}_3$ so that $p(0) = 1$.

d) $H = \left\{ \begin{bmatrix} 2s+3t \\ s-2t \\ 5t \end{bmatrix} : s, t \text{ in } \mathbb{R} \right\}$.

2. (15) Let $\mathcal{B} = \{[1\ 2]^T, [0\ 1]^T\}$ and $\mathcal{C} = \{[0\ 1]^T, [1\ 0]^T\}$ be two bases of $\mathbb{R}^2$. Find the coordinate change matrix $\underset{\mathcal{B}\leftarrow\mathcal{C}}{P}$ from $\mathcal{C}$ to $\mathcal{B}$ coordinates.

3. (15) Solve the equation $\begin{bmatrix} I_5 & X \\ 0 & I_5 \end{bmatrix} \begin{bmatrix} A \\ Y \end{bmatrix} = \begin{bmatrix} B \\ C \end{bmatrix}$ for X and Y in terms of A, B, and C. Assume that A, B, and C are invertible 5×5 matrices.

4. (10) Determine whether or not $\{2t^2,\ (t-2)^2,\ t-1\}$ is a basis for $\mathbb{P}_2$. If it is a basis, find the coordinate vector of $p(t) = t+1$ relative to this basis.

5. (30) Indicate whether each statement is true or false.

a) $\text{rank}(A) = \text{rank}(A^T)$.

b) If A is invertible, then $\text{rank}(A) = \text{rank}(A^{-1})$.

c) Any four vectors which span a four dimensional vector space V form a basis for V.

d) If v_1, v_2, v_3 are linearly independent vectors in a four dimensional vector space V, then there is a vector v_4 so that v_1, v_2, v_3, v_4 is a basis for V.

e) Every vector space has a basis.

f) The vector spaces $\mathbb{M}_{2\times 3}$ and $\mathbb{P}_5$ are isomorphic.

g) $\det(2A) = 2\det(A)$.

h) $\det(A) = \det(A^T)$.

i) If $T: V \to W$ is a linear transformation, then the kernel of T is a subspace of V.

j) If $T: V \to W$ is a linear transformation, then the kernel of T is a subspace of W.

6. (10) Short answer.

a) If A is an $m \times n$ matrix, then $\text{rank}(A) + \dim Nul(A) =$

b) If B is obtained from A by switching two rows of A, then $\det(B) =$

7. (20) Let $T: V \to W$ be a linear transformation. Given a subspace U of V, let $T(U)$ denote the set of all images of the form $T(x)$, where x is in U. Show that $T(U)$ is a subspace of W.