MATH 461 FINAL EXAM Problem 1 May 18, 2005

Name: TA: Section:

Let $A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & -1 \\ 1 & 1 & 0 \end{bmatrix}$ and $\mathbf{b} = \begin{bmatrix} 4 \\ 2 \\ 0 \end{bmatrix}$.

1a) [20] Find all least squares solutions to $A\mathbf{x} = \mathbf{b}$.

1b) [5] Find the orthogonal projection of $\mathbf{b}$ to the column space of A.

HONOR PLEDGE: I pledge on my honor that I have not given or received any unauthorized assistance on this examination.

____

____

Signature ____________________________________

MATH 461 FINAL EXAM Problem 2 May 18, 2005

Name: TA: Section:

$$\text{Let } A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 3 & 6 & 0 \end{bmatrix}$$

2a) [10] Find the reduced echelon form of A.

2b) [3] Is A symmetric?______

2c) [3] What is the rank of A?______

2d) [3] Is A invertible?______

2e) [3] Is A diagonalizable?______

2f) [10] Find an orthogonal basis for the column space of A.

2g) [10] Find an orthonormal basis for the null space of A.

MATH 461 FINAL EXAM Problem 3 May 18, 2005

Name: TA: Section:

Let A be the matrix of the quadratic form $-x_1^2 + x_3^2 + 4x_1x_2 + 4x_2x_3$.

3a) [10] Find the characteristic polynomial of A.

3b) [15] Find the eigenvalues and eigenvectors of A. You should be able to find eigenvectors with all integer entries.

3c) [5] Is the quadratic form positive definite?______

3d) [5] Find an orthogonal matrix P such that the change of variable $x = Py$ transforms $x^T Ax$ into a quadratic form with no cross-product terms.

MATH 461 FINAL EXAM Problem 4 May 18, 2005

Name: TA: Section:

Let V be the span of $\{t^2 - t, t - 1, t^3 - 1\}$ in $\mathbb{P}_3$.

4a) [10] Show that $\mathcal{B} = \{t^2 - t, t - 1, t^3 - 1\}$ is a basis of V.

4b) [15] Let $T: V \to \mathbb{R}^3$ be the linear transformation defined by $T(p) = \begin{bmatrix} p(1) \\ 2p(0) \\ p(2) \end{bmatrix}$. Let $\mathcal{C} = \{2e_2 - e_3, e_3, e_1\}$. Find the matrix for T relative to the bases $\mathcal{B}$ and $\mathcal{C}$.

4c) [5] Find the kernel of T.

MATH 461 FINAL EXAM Problem 5 May 18, 2005

Name: TA: Section:

For each of the following sets H, determine whether or not H is a subspace (and give adequate reasons for your answer). If it is a subspace, find a basis if possible and determine the dimension of H.

5a) [8] H is the set of $[x\ y\ z]^T$ in $\mathbb{R}^3$ so that $x + 2y + 4z = 0$.

5b) [8] H is the set of singular 2×2 matrices in $\mathbb{M}_{2\times 2}$.

5c) [8] H is the set of polynomials $p(t)$ in $\mathbb{P}_2$ so that $\int_{-1}^{1} p(t)\, dt = 2p(0)$.

5d) [8] H is the span of $\{e_1, e_1 + e_3, e_1 - e_3\}$ in $\mathbb{R}^3$.

MATH 461 FINAL EXAM Problem 6 May 18, 2005

Name: TA: Section:

Mark each statement True (always true), False (always false), Maybe (sometimes true or sometimes false, depending on A, S, etc.), or fill in the short answer. If your answer is Maybe, briefly explain why. Each part is worth 3 points.

a) If V is a finite dimensional inner product space, then V has an orthonormal basis.______

b) If A and B are matrices and $AB = 0$ then either $A = 0$ or $B = 0$.______

c) If a matrix A is orthogonal, then A is nonsingular.______

d) If A and B are 3×3 matrices then $\det(A + B) = \det(A) + \det(B)$.

e) If A is a 5×3 matrix and $Ax = b$ has a solution for all b in $\mathbb{R}^5$ then A has a pivot in every ______ .

f) If A is a 7×7 matrix, then A is diagonalizable if and only if $\mathbb{R}^7$ has a basis consisting of eigenvectors of A.______

g) Two eigenvectors of a diagonalizable matrix are orthogonal if they correspond to different eigenvectrors.______

h) If x and y are vectors of equal length in an inner product space, then $x + 2y$ and $2x - y$ are orthogonal.______

i) If the characteristic polynomial of a matrix A has a repeated root, then A is not diagonalizable.______

j) If A is a 4×5 matrix with rank 1, then the dimension of the null space of A is ______ .

k) If A and B are invertible 3×3 matrices, then $(AB)^{-1} =$______ .

l) If A and B are symmetric matrices of the same size, then AB is symmetric.______