Matlab in Math 461, part five

Eigenvalues and Eigenvectors, etc.

If A is a square $n \times n$ matrix, then the command `E = eig(A)` will produce a vector E whose entries are the n eigenvalues of A (including multiplicities). Some eigenvalues may be complex and the real and imaginary parts will be given. To find the eigenvectors, use the command

```
>> [V D] = eig(A)
```

Then D will be a diagonal matrix whose diagonal entries are the eigenvalues (so D = diag(E)) and V will be an $n \times n$ matrix whose columns are the corresponding eigenvectors. In particular, you are guaranteed that $AV = VD$.

If A has n distinct eigenvalues, then we know $\mathbb{R}^n$ has a basis of eigenvectors, so V will be invertible. If there are repeated eigenvalues, however, it may happen that there is not a basis of eigenvectors so V might not be invertible.

Other matlab operations

The trace of a matrix is the sum of its diagonal entries. In matlab just write `trace(A)`. To find the characteristic polynomial of a matrix A, just type `poly(A)` which will return a vector of coefficients of the characteristic polynomial. To find the roots of this polynomial (which are of course the eigenvalues of A) use the matlab command `roots`. The command `roots(poly(A))` is less accurate and more time consuming than the equivalent `eig(A)`. In fact, matlab calculates poly(A) not by evaluating the determinent of $\lambda I - A$ but by first finding eig(A) and then multiplying out the polynomial which has those eigenvalues for roots! Note: the characteristic polynomial calculated by matlab differs slightly from that in the book; it is $\det(\lambda I - A)$ versus $\det(A - \lambda I)$. This results in a sign difference for odd sized matrices.

To find the real part of a matrix or vector X write `real(X)`. To find the imaginary part, write `imag(X)`.

Problems due April 19

Remember to start with `rand('state',sum(100*clock));` so your answers differ from other groups. As usual you may work in groups of two or three but no more.

Problem 1: Let A be a random 6 by 6 matrix with real entries and let B be a random 6 by 6 matrix with complex entries. Find the eigenvalues and eigenvectors of A and B.

a) Lay claims on page 338 that the eigenvalues and eigenvectors of A occur in conjugate pairs. Identify these pairs in your output. (If all your eigenvalues turn out to be real, try again with a different A until you get one with some complex eigenvalues).

b) Do the eigenvalues and eigenvectors of B occur in conjugate pairs? Identify these pairs if they do, otherwise say why they do not.

Problem 2: Generate random complex 3×3, 6×6, and 7×7 matrices and calculate the following quantities: the sum of the eigenvalues, the product of the eigenvalues, the characteristic polynomial, the trace and the determinent. Formulate conjectures on any interrelationships between these things. (Recall the matlab commands sum and prod give the sum and product of a vector's entries).

Problem 3: section 5.5 problem 28. (To check that you entered the matrix correctly, calculate the sum of its columns and rows. Your answers should be `sum(A) = [-0.4000 -1.4000 -1.4000 -1.4000]` and `sum(A') = [-7.4000 -2.8000 -4.6000 10.2000]`.)

Problem 4: Let A be a random 5×5 complex matrix and let `B = A + A'` which is the sum of A and its conjugate transpose. Let `C = A - A'`. Find the eigenvalues of B, C, $\begin{bmatrix} B & C \\ C & B \end{bmatrix}$, $\begin{bmatrix} B & C \\ -C & B \end{bmatrix}$, $\begin{bmatrix} C & B \\ B & C \end{bmatrix}$, and $\begin{bmatrix} C & B \\ -B & C \end{bmatrix}$. Formulate conjectures about your results and any interrelations between them.