Matlab in Math 461, part seven

Least squares solutions

You can find the least squares solution to $Ax = b$ using the methods in Lay, solving the normal equation $A^T Ax = A^T b$. But matlab also finds a least squares solution for you automatically. If A is not a square matrix then `A\b` always finds a least squares solution to $Ax = b$. Matlab may complain if the columns of A are not linearly independent, but generally seems to come up with a good answer nevertheless. However, some care must be taken if A is square and not invertible since then `A\b` will not determine the least squares solution. You can get around this by adding a last row of zeroes to A and b to fool matlab. For example if A is 6×6 you could do `x = [A; zeros(1,6)]\[b; 0]`. Then x will be a least squares solution to $Ax = b$.

Singular value decomposition

The command `[U S V] = svd(A)` finds the singular value decomposition of A. So if A is $m \times n$ then U and V are unitary matrices (which means orthogonal in case A is real), $A = USV^*$, and S is a diagonal $m \times n$ matrix (that is, if A has rank r then S has an $r \times r$ diagonal matrix in its upper left corner and the rest of S is zero). The diagonal entries of S are real and nonnegative, in decreasing order.

Problems due May 10

For all the following problems let A be a random 4×4 complex matrix with rank 2 and nonzero imaginary part. Check that its rank is in fact 2 before proceeding.

Problem 1: Generate a random vector b in $\mathbb{C}^4$. Find a least squares solution $\hat{x}$ to $Ax = b$. You could use the trick given above if you wanted or any other way you prefer. Compute the error vector $b - A\hat{x}$. Check that this error vector is perpendicular to the column space of A. The error is the length $||b - A\hat{x}||$. Check that the error is minimized for the least squares solution by computing the quantity $||b - Ax||$ for several random vectors x and seeing that it is larger than than the error.

Problem 2: Find the singular value decomposition of A. Also compute `orth(A)` and `null(A)`. Based on your results, guess how matlab computes `orth(A)` and `null(A)`.

Problem 3: Find the pseudoinverse A^+ of A (see page 480). Calculate A^+b (using the same b as in problem 1). Exercise 13 on page 492 says that A^+b should be the least squares solution to $Ax = b$ of smallest length. Check that A^+b is a least squares solution by seeing whether its image is the same as $A(\hat{x})$. Compare the length of A^+b with the length of your solution $\hat{x}$ from problem 1. Determine whether or not the length is smaller.