MATH 461 EXAM #1 Problem 1 March 2, 2005

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1. (39) Suppose a matrix A has an echelon form $\begin{bmatrix} 1 & 0 & 3 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ and B has an echelon form $\begin{bmatrix} 2 & 2 & 3 & 0 \\ 0 & 3 & 2 & 0 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 4 \end{bmatrix}$. Answer the following questions. If there is not enough infromation to give an answer, say so.

a) How many pivots does A have? ________

b) How many pivots does B have? ________

c) How many solutions does $Ax = 0$ have? ________

d) How many solutions does $Bx = 0$ have? ________

e) How many solutions does $Ax = [1\ 2\ 3\ 0]^T$ have? ________

f) How many solutions does $Bx = [1\ 2\ 3\ 0]^T$ have? ________

g) Is the linear transformation $x \mapsto Ax$ one to one? ________

h) Is the linear transformation $x \mapsto Bx$ one to one? ________

i) Is the linear transformation $x \mapsto Ax$ onto? ________

j) Is the linear transformation $x \mapsto Bx$ onto? ________

k) Which of A or B is invertible? ________

l) One solution of $Ax = [1\ 2\ 3\ 4]^T$ is $x = [0\ 1\ 0\ 1]^T$. Find all solutions.

m) One solution of $Bx = [1\ 2\ 3\ 4]^T$ is $x = [0\ 1\ 2\ \ -1]^T$. Find all solutions.

HONOR PLEDGE: I pledge on my honor that I have not given or received any unauthorized assistance on this examination.

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Signature ____

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2. (36)

a) Suppose A, B, and C are invertible 4×4 matrices. Solve for the 4×4 matrices W, X, Y and Z.

$$\begin{bmatrix} X & Y \\ Z & W \end{bmatrix} \begin{bmatrix} 0 & A \\ B & C \end{bmatrix} = I_8$$

b) Find $\begin{bmatrix} 0 & A \\ B & C \end{bmatrix}^{-1}$, with A, B, and C invertible.

c) Suppose that $T\colon \mathbb{R}^2 \to \mathbb{R}^3$ is a linear transformation and also $T(\begin{bmatrix} 1 \\ 0 \end{bmatrix}) = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ and $T(\begin{bmatrix} 0 \\ 1 \end{bmatrix}) = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$ and $T(\begin{bmatrix} 1 \\ 1 \end{bmatrix}) = \begin{bmatrix} 2 \\ 2 \\ 4 \end{bmatrix}$. Find $T(\begin{bmatrix} 2 \\ -1 \end{bmatrix})$. Also find the standard matrix of T.

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3. (25) Let $\mathbf{v_1} = [1\ 2\ 3\ 1]^T$, $\mathbf{v_2} = [1\ 0\ -1\ 1]^T$, and $\mathbf{v_3} = [1\ 8\ h\ 1]^T$.

a) Find all h so that $\{\mathbf{v_1}, \mathbf{v_2}, \mathbf{v_3}\}$ is linearly dependent.

b) For each h you found in part a), determine if possible weights c_1 and c_2 so that $\mathbf{v_3} = c_1\mathbf{v_1} + c_2\mathbf{v_2}$.

c) For each h you found in part a), determine whether or not $\mathbf{v_3}$ is in Span$\{\mathbf{v_1}, \mathbf{v_2}\}$.