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For each of the following sets H , determine whether or not H is a subspace (and give adequate reasons for your answer). If it is a subspace, find a basis if possible and determine the dimension of H .

1a) [8] H is the set of $[x \ y \ z]^T$ in \mathbb{R}^3 so that $x + 2y + 4z^2 = 0$.

1b) [8] H is the set of diagonal 2×2 matrices in $\mathbb{M}_{2 \times 2}$.

1c) [8] H is the set of polynomials $p(t)$ in \mathbb{P} so that $\int_1^2 p(t) dt = 0$.

HONOR PLEDGE: I pledge on my honor that I have not given or received any unauthorized assistance on this examination.

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Let $T: \mathbb{P}_3 \rightarrow \mathbb{P}_2$ be the transformation $T(p) = p' - p(0)$, so for example $T(t^3 - t + 2) = 3t^2 - 1 - 2 = 3t^2 - 3$.

2a) [8] Show that T is a linear transformation.

2b) [6] Find a basis for the kernel of T .

2c) [6] Find the dimension of the kernel of T and dimension of the range of T .

2d) [6] Is T one to one?_____ Is T onto?_____ Give reasons for your answers below.

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Let $A = \begin{bmatrix} 1 & 2 & 3 & 9 \\ 2 & 0 & -2 & 6 \\ 0 & 1 & 2 & 3 \\ 1 & 2 & 3 & 7 \end{bmatrix}$. You may use the following matlab output in this question:

```
EDU>A = [1 2 3 9; 2 0 -2 6; 0 1 2 3; 1 2 3 7];
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```
EDU>rref(A)
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ans =
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1 0 -1 0
0 1 2 0
0 0 0 1
0 0 0 0
```

3a) [6] What is the rank of A ?_____ What is the rank of A^T ?_____

3b) [6] Find a basis of the column space of A .

3c) [6] Find a basis of the row space of A .

3d) [8] Find a basis of the null space of A .

3e) [4] What is the determinant of A ?_____

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4a) [6] Is $\left\{ \begin{bmatrix} 1 & 2 \\ 1 & 2 \end{bmatrix}, \begin{bmatrix} 1 & 4 \\ 0 & 0 \end{bmatrix} \right\}$ a linearly independent set in the vector space $\mathbb{M}_{2 \times 2}$ of 2×2 matrices? You must give an adequate reason for your answer.

4b) [6] Is $\{1, \cos^2(t), \sin^2(t)\}$ a linearly independent set of functions defined on \mathbb{R} ? You must give an adequate reason for your answer.

4c) [8] Find a basis \mathcal{B} for the span of $\{t^2, t - 1, t^2 + 2t - 2\}$ in \mathbb{P}_2 . Find the coordinates of $t^2 + 5t - 5$ relative to your basis \mathcal{B} .