MATH 461 EXAM # 3 Problem 1 April 29, 2005

Name: TA: Section:

Let $A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 2 \end{bmatrix}$ and $\mathbf{b} = \begin{bmatrix} 6 \\ 0 \\ 0 \end{bmatrix}$.

1a) [20] Find all least squares solutions to $A\mathbf{x} = \mathbf{b}$.

1b) [5] Find the orthogonal projection of $\mathbf{b}$ to the column space of A.

HONOR PLEDGE: I pledge on my honor that I have not given or received any unauthorized assistance on this examination.

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Signature ______________________________________

MATH 461 EXAM # 3 Problem 2 April 29, 2005

Name: TA: Section:

Consider the vector space $C[0,1]$ of continuous real valued functions on the interval $[0,1]$ with inner product $\langle f, g\rangle = \int_0^1 f(t)g(t)\,dt$. Let H be the subspace spanned by the functions 1 and t. Let $T\colon H \to H$ be differentiation, $T(f) = df/dt$.

2a) [10] Find an orthonormal basis $\mathcal{B}$ for H.

2b) [10] Find the matrix of T relative to the basis $\mathcal{B}$ you found in part a).

2c) [5] Find the orthogonal projection of $f(t) = t^3$ to H.

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Suppose there are square matrices A_1, A_2, and A_3. The characteristic polynomial of A_1 is $(t^2 - 1)(t^2 + 2t + 5)$. The characteristic polynomials of A_2 and A_3 are the same, $(t - 1)(t + 6)t^2$. All of the eigenspaces of A_2 have dimension 1. One of the eigenspaces of A_3 has dimension bigger than 1.

3a) [8] Find all the eigenvalues of each A_j.

3b) [6] Which of the A_i are diagonalizable, that is, for which A_i is there a real matrix P so that $P^{-1}A_iP$ is diagonal?

3c) [3] Which eigenspace of A_3 has dimension bigger than 1 and what is its dimension?

3d) [3] Which of the A_i are invertible?

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For each of the following matrices:

+ Find all of its eigenvalues and an eigenvector for each eigenvalue.
+ If possible, find a (possibly complex) matrix P and a diagonal matrix D so that the given matrix equals PDP^{-1}.
+ If possible, find a real matrix Q so that the given matrix is QCQ^{-1} where C is of the form $C = \begin{bmatrix} a & -b \\ b & a \end{bmatrix}$.

4a) [15] $\begin{bmatrix} 4 & -1 \\ 4 & 0 \end{bmatrix}$

4b) [15] $\begin{bmatrix} 4 & -1 \\ 5 & 0 \end{bmatrix}$