Math 340
HW #2
September 13, 2006

1.6: 2, 6, 8, 10
1.7: 1, 2, 3, 4, 6, 7
1.8: 2, 3b, 7, 8
1.9: 4

Solutions

1.6 #2

$$C = \left[\begin{array}{cc|ccc} 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ \hline 0 & 0 & 2 & 3 & 1 \\ 0 & 0 & 5 & 0 & 2 \\ 0 & 0 & 4 & 1 & 2 \end{array}\right] \qquad D = \left[\begin{array}{cc|ccc} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ \hline 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

$$C = \begin{bmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{bmatrix} \qquad D = \begin{bmatrix} D_{11} & D_{12} \\ D_{21} & D_{22} \end{bmatrix}$$

$$CD = \begin{bmatrix} C_{11}D_{11} + C_{12}D_{21} & C_{11}D_{12} + C_{12}D_{22} \\ C_{21}D_{11} + C_{22}D_{21} & C_{21}D_{12} + C_{22}D_{22} \end{bmatrix} \qquad (*)$$

$$C_{11}D_{11} = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \qquad C_{12}D_{21} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$C_{21}D_{11} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \qquad C_{22}D_{21} = \begin{bmatrix} 2 & 3 & 1 \\ 5 & 0 & 2 \\ 4 & 1 & 2 \end{bmatrix}\begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$C_{11}D_{12} = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \qquad C_{12}D_{22} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$C_{21}D_{12} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \qquad C_{22}D_{22} = \begin{bmatrix} 2 & 3 & 1 \\ 5 & 0 & 2 \\ 4 & 1 & 2 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 3 & 1 \\ 5 & 0 & 2 \\ 4 & 1 & 2 \end{bmatrix}$$

$$CD = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2 & 3 & 1 \\ 0 & 0 & 5 & 0 & 2 \\ 0 & 0 & 4 & 1 & 2 \end{bmatrix}$$

You don't have to include all these steps, but please show that you know how the partiion works. Including (*) would be good.

1.6 #6

$$A = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}$$

$$A^T = \begin{bmatrix} A_{11}^T & A_{21}^T \\ A_{12}^T & A_{22}^T \end{bmatrix}$$

You can tell you need to take the transpose of each element matrix of A (e.g. A_{11}, A_{12}) by writing out an example. Try C from problem 2.

from thm 1.13

$$A = \begin{bmatrix} A_{11} & A_{12} \\ 0 & A_{22} \end{bmatrix} \qquad A^{-1} = \begin{bmatrix} A_{11}^{-1} & -A_{11}^{-1} A_{12} A_{22}^{-1} \\ 0 & A_{22}^{-1} \end{bmatrix}$$

Find: $\begin{bmatrix} A_{11} & 0 \\ A_{21} & A_{22} \end{bmatrix}^{-1}$

$$\begin{bmatrix} A_{11} & 0 \\ A_{21} & A_{22} \end{bmatrix}^T = \begin{bmatrix} A_{11}^T & A_{21}^T \\ 0 & A_{22}^T \end{bmatrix}$$

So to find: $\begin{bmatrix} A_{11} & 0 \\ A_{21} & A_{22} \end{bmatrix}^{-1}$ find: $\begin{bmatrix} A_{11}^T & A_{21}^T \\ 0 & A_{22}^T \end{bmatrix}^{-1} = \begin{bmatrix} (A_{11}^T)^{-1} & (-A_{11}^T)^{-1}(A_{21}^T)(A_{22}^T)^{-1} \\ 0 & (A_{22}^T)^{-1} \end{bmatrix}$

Then take the transpose: $\begin{bmatrix} (A_{11}^T)^{-1} & (-A_{11}^T)^{-1}(A_{21}^T)(A_{22}^T)^{-1} \\ 0 & (A_{22}^T)^{-1} \end{bmatrix}^T = \begin{bmatrix} A_{11}^{-1} & 0 \\ -A_{22}^{-1} A_{21} A_{11}^{-1} & A_{22}^{-1} \end{bmatrix}$

Check: $\begin{bmatrix} A_{11} & 0 \\ A_{21} & A_{22} \end{bmatrix} \begin{bmatrix} A_{11}^{-1} & 0 \\ -A_{22}^{-1} A_{21} A_{11}^{-1} & A_{22}^{-1} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ A_{21}A_{11}^{-1} + A_{22}(-A_{22}^{-1})A_{21}A_{11}^{-1} & 1 \end{bmatrix}$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Note: you don't have to do the check, but it could save you points!

So $\begin{bmatrix} A_{11} & 0 \\ A_{21} & A_{22} \end{bmatrix}^{-1} = \begin{bmatrix} A_{11}^{-1} & 0 \\ -A_{22}^{-1} A_{21} A_{11}^{-1} & A_{22}^{-1} \end{bmatrix}$

1.6 #8

$$A = \begin{bmatrix} 1 & 2 & -1 & 0 \\ 4 & 0 & 2 & 1 \\ 2 & -5 & 1 & 2 \end{bmatrix} \qquad B = \begin{bmatrix} 3 & 1 & 2 \\ -4 & 5 & -2 \\ 1 & 0 & 3 \\ 2 & 3 & -1 \end{bmatrix}$$

Thm 1.11 : $\text{Row}_i(AB) = a_{i1}\text{Row}_1(B) + a_{i2}\text{Row}_2(B) + \dots + a_{in}\text{Row}_n(B)$

$$\begin{aligned} \text{Row}_3(AB) &= a_{31}\text{Row}_1(B) + a_{32}\text{Row}_2(B) + \dots + a_{3n}\text{Row}_n(B). \\ (*) \quad &= 2(3\ 1\ 2) + (-5)(-4\ 5\ -2) + 1(1\ 0\ 3) + 2(2\ 3\ -1). \\ &= (6\ 2\ 4) + (20\ -25\ 10) + (1\ 0\ 3) + (4\ 6\ -2) \\ &= (31\ -17\ 15). \end{aligned}$$

I didn't take off any points if you stopped at (*), since the question only asked for a linear combination.

Thm 1.12 : $\text{Col}_j(AB) = \text{Col}_1(A)b_{1j} + \text{Col}_2(A)b_{2j} + \dots + \text{Col}_n(A)b_{nj}$.

$$\begin{aligned} \text{Col}_2(AB) &= \text{Col}_1(A)b_{12} + \text{Col}_2(A)b_{22} + \text{Col}_3(A)b_{23} + \text{Col}_4(A)b_{24} \\ &= \begin{pmatrix}1\\4\\2\end{pmatrix}1 + \begin{pmatrix}2\\0\\-5\end{pmatrix}5 + \begin{pmatrix}-1\\2\\1\end{pmatrix}0 + \begin{pmatrix}0\\1\\2\end{pmatrix}3 \\ &= \begin{pmatrix}1\\4\\2\end{pmatrix} + \begin{pmatrix}10\\0\\-25\end{pmatrix} + \begin{pmatrix}0\\3\\6\end{pmatrix} \\ &= \begin{pmatrix}11\\7\\-17\end{pmatrix}. \end{aligned}$$

Note: $\text{Col}_j(A)$ is a <u>column</u> vector. It should be of the form $\begin{pmatrix}a\\b\\c\end{pmatrix}$.

Also, since our b_{ij} are scalar, we know that scalar multiplication of a matrix commutes (eg: $5\begin{pmatrix}2\\0\\-5\end{pmatrix} = \begin{pmatrix}2\\0\\-5\end{pmatrix}5$).

However if B were partitioned, and b_{ij} were a matrix, then it would not commute. Learning the order of the theorem (to right-multiply each column) will serve all eventualities.

1.6 #10

$$B = \begin{bmatrix} A & I & 0 \\ I & A & I \\ 0 & I & A \end{bmatrix}$$ Find B^2 and B^3.

Almost everyone just did the matrix multiplication BB and BBB.

To use something from this section, I tried partitioning the matrix to find B^2:

$$B = \left[\begin{array}{cc|c} A & I & 0 \\ I & A & I \\ \hline 0 & I & A \end{array}\right] = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \qquad B^2 = \begin{bmatrix} A^2+BC & AB+BD \\ CA+DC & CB+D^2 \end{bmatrix}.$$

$$A^2 = \begin{pmatrix} A & I \\ I & A \end{pmatrix}^2 = \begin{pmatrix} A^2+I & 2A \\ 2A & A^2+I \end{pmatrix} \qquad BC = \begin{pmatrix} 0 \\ I \end{pmatrix}(0\ I) = \begin{pmatrix} 0 & 0 \\ 0 & I \end{pmatrix}$$

$$A^2 + BC = \begin{pmatrix} A^2+I & 2A \\ 2A & A^2+2I \end{pmatrix}$$

$$AB = \begin{pmatrix} A & I \\ I & A \end{pmatrix}\begin{pmatrix} 0 \\ I \end{pmatrix} = \begin{pmatrix} I \\ A \end{pmatrix} \qquad BD = \begin{pmatrix} 0 \\ I \end{pmatrix}(A) = \begin{pmatrix} 0 \\ A \end{pmatrix}. \qquad AB + BD = \begin{pmatrix} I \\ 2A \end{pmatrix}$$

$$CA = (0\ I)\begin{pmatrix} A & I \\ I & A \end{pmatrix} = (I\ A) \qquad DC = (A)(0\ I) = (0\ A) \qquad CA + DC = (I\ 2A).$$

$$CB = (0\ I)\begin{pmatrix} 0 \\ I \end{pmatrix} = I \qquad D^2 = A^2 \qquad CB + D^2 = I + A^2.$$

So $$B^2 = \begin{bmatrix} A^2+I & 2A & I \\ 2A & A^2+2I & 2A \\ I & 2A & A^2+I \end{bmatrix}.$$

$$B^3 = B^2B = \begin{bmatrix} A^3+3A & 3A^2+2I & 3A \\ 3A^2+2I & 6A+A^3 & 3A^2+2I \\ 3A & 2I+3A^2 & A^3+3A \end{bmatrix}$$

I didn't show any work here for B^3 because most of you did fine on these. But there were a lot of calculation errors. The more work you show, the more likely I am to be able to find your mistake + give partial credit.

1.7 #1

Want to show: the set of all $n\times n$ upper triangular matrices is closed with respect to matrix addition and matrix multiplication

Let T_1 and T_2 be $n\times n$ upper triangular.

$$T_1 = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ 0 & a_{22} & & \vdots \\ 0 & 0 & \ddots & \vdots \\ \vdots & & & \\ 0 & 0 & 0 \cdots & a_{nn} \end{bmatrix} \quad T_2 = \begin{bmatrix} b_{11} & b_{12} & \cdots & b_{1n} \\ 0 & b_{22} & \cdots & \\ 0 & 0 & \ddots & \\ \vdots & & & \\ 0 & 0 & \cdots & b_{nn} \end{bmatrix} \quad T_1+T_2 = \begin{bmatrix} a_{11}+b_{11} & a_{12}+b_{12} & \cdots & a_{1n}+b_{1n} \\ 0+0 & a_{22}+b_{22} & \cdots & \\ 0+0 & 0+0 & & \\ \vdots & & & \\ 0+0 & 0+0 & \cdots & a_{nn}+b_{nn} \end{bmatrix}$$

Therefore T_1+T_2 is upper triangular.

$$T_1T_2 = \begin{bmatrix} a_{11}b_{11}+a_{12}\cdot 0+\cdots+a_{1n}\cdot 0 & a_{11}\cdot b_{12}+a_{12}\cdot b_{22}+a_{13}\cdot 0+\cdots+a_{1n}\cdot 0 & \cdots & a_{11}b_{1n}+\cdots+a_{1n}b_{1n} \\ 0\cdot b_{11}+a_{22}\cdot 0+\cdots+a_{2n}\cdot 0 & 0\cdot b_{12}+a_{22}\cdot b_{22}+a_{23}\cdot 0+\cdots+a_{2n}\cdot 0 & \cdots & 0\cdot b_{1n}+\cdots+a_{nn}b_{1n} \\ \vdots & & & \vdots \\ 0\cdot b_{11}+0\cdot 0+\cdots+0\cdot 0+a_{nn}\cdot 0 & 0\cdot b_{12}+0\cdot b_{22}+\cdots+0\cdot 0+a_{nn}b_{2n} & \cdots & 0\cdot b_{1n}+\cdots+0\,b_{n-1n}+0\cdot b_{nn} \end{bmatrix}$$

For any entry in T_1T_2, $\text{ent}_{ij} = \text{row}_i(T_1)\,\text{col}_j(T_2)$
$= a_{i1}\cdot b_{1j} + a_{i2}b_{2j} + \cdots + a_{in}b_{nj}$

We know that since T_1 and T_2 are upper triangular, $\text{ent}_{kl}(T_1) = 0$ if $k>l$ and $\text{ent}_{kl}(T_2)=0$ if $k>l$.

We want to show $\text{ent}_{ij}(T_1T_2)=0$ if $i>j$. So let $i>j\geq 1$. Then,

For any term in ent_{ij}, $a_{ir}b_{rj}$, either $i>r$ or $r>j$.

But we know that $a_{ir}=0$ if $i>r$ and $b_{rj}=0$ if $r>j$.

Therefore $a_{ir}\cdot b_{rj} = 0\cdot b_{rj}$ or $a_{ir}b_{rj} = a_{ir}0$. so $a_{ir}b_{rj}=0$ whenever $i>j$. We know $\text{ent}_{ij} = a_{i1}b_{1j}+\cdots+a_{in}b_{nj}$ so each term is 0. $\text{ent}_{ij} = 0+0+\cdots+0 = 0$.

Therefore T_1T_2 is upper triangular.

So the set $U = \{\text{upper triangular } n\times n \text{ matrices}\}$ is closed under addition and multiplication.

The diagonal entries of T_1+T_2 are $\text{ent}_{ii} = a_{ii}+b_{ii}$
of T_1T_2 are $\text{ent}_{ii} = a_{ii}b_{ii}$

Let $p(x) = a_0 + a_1 x + a_2 x^2 + \dots + a_m x^m$

Let T bean $n \times n$ upper triangular matrix.

Then $p(T) = a_0 I + a_1 T + a_2 T^2 + \dots + a_m T^m$

because we have shown that upper triangular matrices are closed under matrix multiplication, T^s is upper triangular, for $s > 0$.

Each coefficient of a power of T is a scalar, and multiplying an upper triangular by a scalar yields an upper triangular matrix.

Then the polynomial is a sum of upper triangular matrices, so we know that $p(T)$ is upper triangular. (because closed under addition).

Diagonal Entries:

$$\begin{aligned}
\mathrm{ent}_{ii}(p(T)) &= \mathrm{ent}_{ii}(a_0 I) + \mathrm{ent}_{ii}(a_1 T) + \dots + \mathrm{ent}_{ii}(a_m T^m) \\
&= a_0 + a_1 \cdot \mathrm{ent}_{ii}(T) + a_2 \cdot \mathrm{ent}_{ii}(T^2) + \dots + a_m \cdot \mathrm{ent}_{ii}(T^m) \\
&= a_0 + a_1 \cdot \mathrm{ent}_{ii}(T) + a_2 \cdot (\mathrm{ent}_{ii}(T))^2 + \dots + a_m \cdot (\mathrm{ent}_{ii}(T))^m \\
&= p(t_{ii}).
\end{aligned}$$

1.7 #2

Show that A^TA is symmetric with nonnegative diagonal entries.

Note: to show a matrix is symmetric, start by trying to show it equals its transpose directly.

$(A^TA)^T = A^T A^{T^T} = A^TA$. Therefore A^TA is symmetric.

A is $m \times n$, so A^TA is $n \times n$.

for $1 \le i \le n$, $\text{ent}_{ii}(A^TA) = \text{row}_i(A^T) \cdot \text{col}_i(A)$

$$= \text{col}_i(A) \cdot \text{col}_i(A)$$

$$= a_{1i} \cdot a_{1i} + a_{2i} \cdot a_{2i} + \dots + a_{2n} \cdot a_{2n}$$

$$= a_{1i}^2 + a_{2i}^2 + \dots + a_{2n}^2$$

Since the diagonal entries are the sum of squares, they are non-negative.

Important note:

Many people tried to give specific examples for this and other problems.

A specific example only stands if it is a counter example.

Otherwise, a general proof must be given.

1.7 #3

Let A be a Hermitian matrix. Then $A^* = A$.

for $1 \le j \le n$

$$\text{ent}_{jj}(A^*) = \text{ent}_{jj}(\overline{A^T})$$

$$= \overline{\text{ent}_{jj}(A^T)} \quad \text{See definition 1.11 on page 29.}$$

$$= \overline{\text{ent}_{jj}(A)}$$ because A^T and A have the same diagonal entries.

Since we know $A^* = A$, then $\text{ent}_{jj}(A^*) = \overline{\text{ent}_{jj}(A)}$. tells us that $\text{ent}_{jj}(A) = \overline{\text{ent}_{jj}(A)}$. If the conjugate of a complex number is equal to the number, then it is real.

($a+bi = a-bi$ implies $b=0$).

Therefore the diagonal entries of A are real.

Let A be a skew Hermitian matrix. Then $A^* = -A$.

$$\text{ent}_{jj}(A^*) = -\text{ent}_{jj}(A)$$

$$\overline{\text{ent}_{jj}(A)} = -\text{ent}_{jj}(A)$$

Let $\text{ent}_{jj}(A) = a+bi$

then $\overline{a+bi} = -(a+bi)$

$$a-bi = -a-bi$$

$$a = -a$$

$$a = 0.$$

So $\text{ent}_{jj}(A) = bi$ for some $b \in \mathbb{R}$.

Therefore the diagonal entries of A are purely imaginary.

1.7 #4

A is symmetric
Want to show that P^TAP is symmetric for any compatible choice of P.

To show P^TAP is symmetric we need to show that $(P^TAP)^T = P^TAP$

$(P^TAP)^T = P^TA^TP^{T^T} = P^TA^TP$ we know that $A^T = A$ (given)

so $P^TA^TP = P^TAP$.

Therefore $(P^TAP)^T = P^TAP$ so P^TAP is symmetric.

1.7 #6

Let A be an $n \times n$ triangular matrix such that $A^T = A$.
Therefore $\text{ent}_{ij}(A) = \text{ent}_{ji}(A)$.
Let $i \neq j$. Then $\text{ent}_{ij}(A) = 0$ or $\text{ent}_{ji}(A) = 0$ since A is triangular.
Then $\text{ent}_{ij}(A) = 0$ for all $i \neq j$. since A is symmetric.
Therefore only the diagonal of A can have non zero entries.

1.8 #2

$$A = \begin{bmatrix} 0 & 1 & 2 \\ -1 & 3 & 0 \\ 1 & -2 & 1 \end{bmatrix} \xrightarrow{r_1 \leftrightarrow r_3} \begin{bmatrix} 1 & -2 & 1 \\ -1 & 3 & 0 \\ 0 & 1 & 2 \end{bmatrix} \xrightarrow{r_2 = r_2 + r_1} \begin{bmatrix} 1 & -2 & 1 \\ 0 & 1 & 1 \\ 0 & 1 & 2 \end{bmatrix} \xrightarrow{r_3 = r_3 - r_2} \begin{bmatrix} 1 & -2 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\xrightarrow{r_2 = r_2 - r_3} \begin{bmatrix} 1 & -2 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \xrightarrow{r_1 = r_1 + 2r_2 - r_3} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$B = \begin{bmatrix} 3 & -1 & 2 & 1 & 0 & 0 \\ 2 & 1 & 1 & 0 & 1 & 0 \\ 1 & -3 & 0 & 0 & 0 & 1 \end{bmatrix} \xrightarrow{r_1 = r_3;\, r_3 = r_1} \begin{bmatrix} 1 & -3 & 0 & 0 & 0 & 1 \\ 2 & 1 & 1 & 0 & 1 & 0 \\ 3 & -1 & 2 & 1 & 0 & 0 \end{bmatrix} \xrightarrow[r_3 = r_3 - 3r_1]{r_2 = r_2 - 2r_1} \begin{bmatrix} 1 & -3 & 0 & 0 & 0 & 1 \\ 0 & 7 & 1 & 0 & 1 & -2 \\ 0 & 8 & 2 & 1 & 0 & -3 \end{bmatrix}$$

$$\xrightarrow[r_3 = r_3/8]{r_2 = r_2/7} \begin{bmatrix} 1 & -3 & 0 & 0 & 0 & 1 \\ 0 & 1 & 1/7 & 0 & \frac{1}{7} & -2/7 \\ 0 & 1 & \frac{1}{4} & \frac{1}{8} & 0 & -\frac{3}{8} \end{bmatrix} \xrightarrow{r_3 = r_3 - r_2} \begin{bmatrix} 1 & -3 & 0 & 0 & 0 & 1 \\ 0 & 1 & 1/7 & 0 & \frac{1}{7} & -\frac{2}{7} \\ 0 & 0 & \frac{3}{28} & \frac{1}{8} & -\frac{1}{7} & -\frac{5}{56} \end{bmatrix} \xrightarrow{r_3 = 28r_3} \begin{bmatrix} 1 & -3 & 0 & 0 & 0 & 1 \\ 0 & 1 & 1/7 & 0 & \frac{1}{7} & -\frac{2}{7} \\ 0 & 0 & 3 & \frac{7}{2} & -4 & -\frac{5}{2} \end{bmatrix}$$

$$\xrightarrow{r_3 = \frac{r_3}{3}} \begin{bmatrix} 1 & -3 & 0 & 0 & 0 & 1 \\ 0 & 1 & \frac{1}{7} & 0 & \frac{1}{7} & -\frac{2}{7} \\ 0 & 0 & 1 & \frac{7}{6} & -\frac{4}{3} & -\frac{5}{6} \end{bmatrix} \xrightarrow{r_2 = r_2 - \frac{r_3}{7}} \begin{bmatrix} 1 & -3 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & -\frac{1}{6} & \frac{1}{3} & -\frac{1}{6} \\ 0 & 0 & 1 & 7/6 & -\frac{4}{3} & -\frac{5}{6} \end{bmatrix} \xrightarrow{r_1 = r_1 + 3r_2} \begin{bmatrix} 1 & 0 & 0 & -\frac{1}{2} & 1 & \frac{1}{2} \\ 0 & 1 & 0 & -\frac{1}{6} & \frac{1}{3} & -\frac{1}{6} \\ 0 & 0 & 1 & \frac{7}{6} & -\frac{4}{3} & -\frac{5}{6} \end{bmatrix}$$

I tried to show a lot more steps than I normally would (or you have to) to make sure this was clear.

1.8 #3b

$$3x - y + 2z = 2$$
$$2x + y + z = -1$$
$$x + 3y = 2$$

$$\left[\begin{array}{ccc|c} 3 & -1 & 2 & 2 \\ 2 & 1 & 1 & -1 \\ 1 & 3 & 0 & 2 \end{array}\right] \longrightarrow \left[\begin{array}{ccc|c} 1 & 3 & 0 & 2 \\ 0 & -5 & 1 & -5 \\ 0 & -10 & 2 & -4 \end{array}\right]$$

$$\longrightarrow \left[\begin{array}{ccc|c} 1 & 3 & 0 & 2 \\ 0 & -5 & 1 & -5 \\ \boxed{0} & \boxed{0} & \boxed{0} & \boxed{6} \end{array}\right]$$

There are no solutions to this system.

Be sure to follow directions. If the problem asks for a specific process or approach to solve, please use *that* approach or process.

1.8 #7

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 5 \\ 3 & 5 & 6 \end{bmatrix} \qquad k_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \qquad k_2 = \begin{bmatrix} 1 \\ -3 \\ 2 \end{bmatrix} \qquad k_3 = \begin{bmatrix} 1 \\ 2 \\ -2 \end{bmatrix}$$

$$[A \,\vdots\, k_1 \,\vdots\, k_2 \,\vdots\, k_3] = \left[\begin{array}{ccc|c|c|c} 1 & 2 & 3 & 1 & 1 & 1 \\ 2 & 4 & 5 & 1 & -3 & 2 \\ 3 & 5 & 6 & 1 & 2 & -2 \end{array}\right] \longrightarrow \begin{bmatrix} 1 & 2 & 3 & 1 & 1 & 1 \\ 0 & 0 & -1 & -1 & -5 & 0 \\ 0 & -1 & -3 & -2 & -1 & -5 \end{bmatrix}$$

$$\longrightarrow \begin{bmatrix} 1 & 0 & -3 & -3 & -1 & -9 \\ 0 & 0 & 1 & 1 & 5 & 0 \\ 0 & 1 & 3 & 2 & 1 & 5 \end{bmatrix} \longrightarrow \begin{bmatrix} 1 & 0 & -3 & -3 & -1 & -9 \\ 0 & 1 & 0 & -1 & -14 & 5 \\ 0 & 0 & 1 & 1 & 5 & 0 \end{bmatrix}$$

$$\longrightarrow \left[\begin{array}{ccc|c|c|c} 1 & 0 & 0 & 0 & 14 & -9 \\ 0 & 1 & 0 & -1 & -14 & 5 \\ 0 & 0 & 1 & 1 & 5 & 0 \end{array}\right]$$

for $Ax = k_1 \quad x = \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ -1 \\ 1 \end{pmatrix}$

$Ax = k_2 \quad x = \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 14 \\ -14 \\ 5 \end{pmatrix}$

$Ax = k_3 \quad x = \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -9 \\ 5 \\ 0 \end{pmatrix}$

You need to show me enough work that I know you solved the problem, but do make use of the tools at hand. The answer to this problem is in the back of the book. You can catch errors by checking your answer against theirs.

1.8 #8

$$A = \begin{bmatrix} 1 & 3 & 3 \\ 1 & 3 & 4 \\ 1 & 4 & 3 \end{bmatrix} \qquad Ax = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \qquad Ay = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \qquad Az = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$\left[\begin{array}{ccc|c|c|c} 1 & 3 & 3 & 1 & 0 & 0 \\ 1 & 3 & 4 & 0 & 1 & 0 \\ 1 & 4 & 3 & 0 & 0 & 1 \end{array}\right] \longrightarrow \begin{bmatrix} 1 & 3 & 3 & 1 & 0 & 0 \\ 0 & 0 & 1 & -1 & 1 & 0 \\ 0 & 1 & 0 & -1 & 0 & 1 \end{bmatrix} \longrightarrow \begin{bmatrix} 1 & 0 & 3 & 4 & 0 & -3 \\ 0 & 1 & 0 & -1 & 0 & 1 \\ 0 & 0 & 1 & -1 & 1 & 0 \end{bmatrix} \longrightarrow \left[\begin{array}{ccc|c|c|c} 1 & 0 & 0 & 7 & -3 & -3 \\ 0 & 1 & 0 & -1 & 0 & 1 \\ 0 & 0 & 1 & -1 & 1 & 0 \end{array}\right]$$

$$x = \begin{pmatrix} 7 \\ -1 \\ -1 \end{pmatrix} \quad y = \begin{pmatrix} -3 \\ 0 \\ 1 \end{pmatrix} \quad z = \begin{pmatrix} -3 \\ 1 \\ 0 \end{pmatrix} \qquad B = (x\ y\ z) = \begin{pmatrix} 7 & -3 & -3 \\ -1 & 0 & 1 \\ -1 & 1 & 0 \end{pmatrix}$$

We can see that our augmented matrix $[A \mid k_1 \mid k_2 \mid k_3]$ where k_1, k_2, k_3 are results of Ax, Ay, Az, respectively, gives us $[A \mid I]$. By reducing this, we get $[I \mid A^{-1}]$ Therefore $AB = AA^{-1} = I$ and $BA = A^{-1}A = I$.

We can check this:

$$AB = \begin{bmatrix} 1 & 3 & 3 \\ 1 & 3 & 4 \\ 1 & 4 & 3 \end{bmatrix} \begin{bmatrix} 7 & -3 & -3 \\ -1 & 0 & 1 \\ -1 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I.$$

$$BA = \begin{bmatrix} 7 & -3 & -3 \\ -1 & 0 & 1 \\ -1 & 1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 3 & 3 \\ 1 & 3 & 4 \\ 1 & 4 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I.$$

1.9 #4

A is nonsingular and symmetric so $A^T = A$.
Want to show that A^{-1} is symmetric

$$(A^{-1})^T = \underbrace{(A^T)}_{=A}{}^{-1} = A^{-1}$$

$(A^{-1})^T = A^{-1}$ so A^{-1} is symmetric.