Math 340
HW Solutions

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1.3 #12

$$\text{proj}_a b = \left(\frac{a \cdot b}{a \cdot a}\right) a$$

$a = i + j + 2k \quad b = 2i - 4j + k$

$$\left(\frac{a \cdot b}{a \cdot a}\right) a = \left(\frac{2 + (-4) + 2}{1 + 1 + 4}\right)(i + j + 2k)$$

$$= \left(\frac{0}{6}\right)(i + j + 2k) = (0,0,0)$$

(28)

(a) Want to show $\|b\| a + \|a\| b$ and $\|b\| a - \|a\| b$ are orthogonal.

suffices to show $(\|b\| a + \|a\| b) \cdot (\|b\| a - \|a\| b) = 0$.

$$(\|b\| a + \|a\| b) \cdot (\|b\| a - \|a\| b)$$

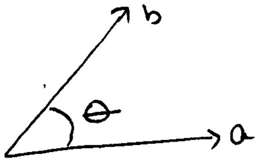
$$= \|b\| a \cdot \|b\| a - \|b\| \cancel{a \cdot \|a\| b + \|a\| b \cdot \|b\| a} - \|a\| b \cdot \|a\| b$$

$$= \|b\|^2 a \cdot a - \|a\|^2 b \cdot b$$

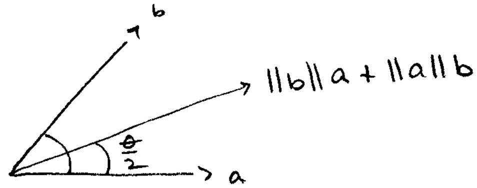
$$= \|b\|^2 \|a\|^2 - \|a\|^2 \|b\|^2$$

$$= 0.$$

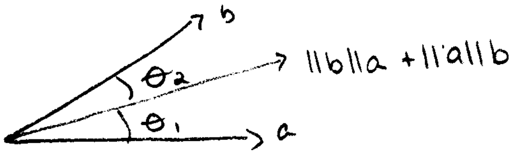
(28)(b) Want to show that $\|b\|a + \|a\|b$ bisects the angle between a and b



wts:



suffices to show that the angle between a and $\|b\|a + \|a\|b$ is equal to the angle between b and $\|b\|a + \|a\|b$, because we know our angle is between 0 and π (page 20 Colley), and all are in the same plane.



$$a \cdot (\|b\|a + \|a\|b) = \|a\| \, \big\| \|b\| a + \|a\|b \big\| \cos\theta_1$$

$$b \cdot (\|b\|a + \|a\|b) = \|b\| \, \big\| \|b\|a + \|a\|b \big\| \cos\theta_2 .$$

$$\cos\theta_1 = \frac{a \cdot (\|b\|a + \|a\|b)}{\|a\| \, \big\|\|b\|a + \|a\|b\big\|} \qquad \cos\theta_2 = \frac{b \cdot (\|b\|a + \|a\|b)}{\|b\| \, \big\|\|b\|a + \|a\|b\big\|}$$

$$\frac{\cos\theta_1}{\cos\theta_2} = \frac{a \cdot (\|b\|a + \|a\|b)}{\|a\| \, \cancel{\big\|\|b\|a + \|a\|b\big\|}} \cdot \frac{\|b\| \, \cancel{\big\|\|b\|a + \|a\|b\big\|}}{b \cdot (\|b\|a + \|a\|b)}$$

Note: I can cancel these because the norm is a scalar!

$$= \frac{(a \cdot (\|b\|a + \|a\|b))\,\|b\|}{\|a\| \, (b \cdot (\|b\|a + \|a\|b))} = \frac{(\|b\|\|a\|^2 + \|a\| a \cdot b)\|b\|}{\|a\| (\|b\| a \cdot b + \|a\| \|b\|^2)}$$

$$= \frac{\|b\|^2 \|a\|^2 + \|a\|\|b\| a \cdot b}{\|b\|^2 \|a\|^2 + \|a\|\|b\| a \cdot b}$$

$$= 1.$$

Therefore, $\cos\theta_1 = \cos\theta_2$ and since $0 \le \theta_1 \le \pi$; $0 \le \theta_2 \le \pi$, we can conclude that $\theta_1 = \theta_2$.

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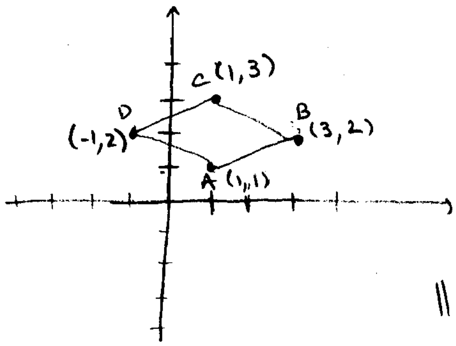
1.4 #10



$A = (1,1)$
$B = (3,2)$
$C = (1,3)$
$D = (-1,2)$.

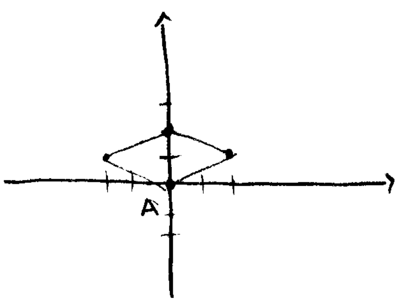
ARea ABCD $= AB \times CD$

$AB = (3-1, 2-1) = (2,1) = 2i + j$
$AD = (-1-1, 2-1) = (-2,1) = -2i + j$

$$AB \times CD = \begin{vmatrix} i & j & k \\ 2 & 1 & 0 \\ -2 & 1 & 0 \end{vmatrix} = 4k$$

$$\|AB \times CD\| = \begin{vmatrix} 2 & 1 \\ -2 & 1 \end{vmatrix} = 2 + 2 = 4 = \text{Area of ABCD.}$$

I could have moved the parallelogram to the origin, and it would still have the same area.



$A = (1,1) - (1,1) = (0,0)$
$B = (3,2) - (1,1) = (2,1)$
$C = (1,3) - (1,1) = (0,2)$
$D = (-1,2) - (1,1) = (-2,1)$

$AB = (2,1) = B$
$AD = (-2,1) = D$.

$$AB \times AD = \begin{vmatrix} 2 & 1 \\ -2 & 1 \end{vmatrix} = 4.$$

(26) (a) vector
(b) nonsense. a.b is a scalar and we have no definition of a scalar dotted with a vector.
(c) nonsense. a.b and c.d are scalars, and we have no definition of cross product for scalars.
(d) scalar. (a×b) is a vector dotted with c
(e) nonsense (a.b) is a scalar, so we can't ×.
(f) vector: (b.c) is a scalar, multiplied by d is a vector crossed with a.
(g) scalar. dot product of two vectors
(h) vector. (a.b) is scalar time c is vector minus a vector.

1.5 #10

Want line through (5,0,6) and perpendicular to the plane $2x-3y+5z=-1$.

The norm of $2x-3y+5z=-1$ is $(2,-3,5)$ so we know that the line we want is parallel to the norm.

Line is $(5,0,6)+(2,-3,5)t$.

$$x = 5+2t$$
$$y = -3t$$
$$z = 6+5t.$$

1.6 #11

We are given $\|a+b\| = \|a-b\|$

Want to show $a \perp b$.

We know it suffices to show $a \cdot b = 0$.

$$\|a+b\| = \|a-b\|$$
$$\sqrt{(a+b)\cdot(a+b)} = \sqrt{(a-b)\cdot(a-b)}$$
$$(a+b)\cdot(a+b) = (a-b)\cdot(a-b)$$
$$a\cdot a + 2a\cdot b + b\cdot b = a\cdot a - 2a\cdot b + b\cdot b$$
$$2a\cdot b = -2a\cdot b$$
$$4a\cdot b = 0$$
$$a\cdot b = 0.$$

Therefore, given $\|a+b\| = \|a-b\|$, a is orthogonal to b.